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SUBROUTINE FM_RK14( A, B, N_ORDER, FM_RK14_F, N_FUNCTION, S, TOL, S1 )

! Solve the vector first-order differential equation s' = f(x,s).

! This routine uses 14th order Runge-Kutta with adjustable step size, starting at x = A
! with state vector S (initial conditions), and returns state vector S1 as the solution
! at x = B.

! N_ORDER is the order of the differential equation (length of vectors S and S1).

! N_FUNCTION is the function number (used in FM_RK14_F) identifying which function f(x,s)
! defines the right-hand-side of the differential equation to be solved.

! TOL is the absolute error tolerance. Because the coefficients are defined with no more than
! about 85-digit precision, TOL should not be less than 1.0e-75.

! The FM precision level should be set to at least 10 digits more than TOL.
! For example, set precision with CALL FM_SET(30) if FM_RK14 will be called with TOL = 1.0e-20.

! The error estimate that is used to control step size will fail if f(x,s) is a function of
! x only, not depending on s. In this case, the differential equation is really just an
! integration problem and should be done as a numerical integration.

USE FMZM
IMPLICIT NONE

INTEGER, PARAMETER :: MAXIMUM_ORDER = 3
INTEGER :: J, N_ORDER, N_FUNCTION
TYPE (FM) :: A, B, TOL, S(MAXIMUM_ORDER), S1(MAXIMUM_ORDER)
TYPE (FM), SAVE :: ERROR_EST, H, TOL2, X
EXTERNAL :: FM_RK14_F
LOGICAL, SAVE :: LAST_STEP

CALL FM_ENTER_USER_ROUTINE

IF (TOL < TO_FM(' 1.0e-75 ')) THEN
  WRITE (*,*) ' '
  WRITE (*,*) ' Error in input to FM_RK14. TOL should not be less than 1.0e-75. It was'
  CALL FM_PRINT(TOL)
  WRITE (*,*) ' '
  STOP
ENDIF

! Pick an initial step size.

TOL2 = ABS(TOL) / 1000
H = ABS(TOL2)**(1.0D0/14)

X = A
LAST_STEP = .FALSE.

DO J = 1, 10**7
  IF (X+H >= B) THEN
    H = B - X

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        LAST_STEP = .TRUE.
    ENDIF
    CALL FM_RK14_STEP( N_ORDER, FM_RK14_F, N_FUNCTION, X, S, H, ERROR_EST, S1 )

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! If the error is too big, try again with halved step size.

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    IF (ERROR_EST > TOL2) THEN
        H = H / 2
        CYCLE
    ENDIF

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! Make the step.

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    IF (LAST_STEP) THEN
        EXIT
    ENDIF
    S(1:N_ORDER) = S1(1:N_ORDER)
    X = X + H

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! If the error is much smaller than TOL2, try doubling the step size.  
 ! Otherwise, if the error is less than TOL2/10, try to fine-tune the  
 ! step size by increasing H slightly.

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    IF (ERROR_EST < TOL2/1.0D+5) THEN
        H = 2*H
    ELSE IF (ERROR_EST < TOL2/10) THEN
        H = 1.05*H
    ENDIF

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ENDDO

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CALL FM_EXIT_USER_ROUTINE
END SUBROUTINE FM_RK14

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SUBROUTINE FM_RK14_STEP( N_ORDER, FM_RK14_F, N_FUNCTION, X, S, H, ERROR_EST, S1 )

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! Do one step of 14th order Runge-Kutta, starting with state vector S(X) and returning  
 ! state vector S1(X+H) after a step of H.  
 ! N\_ORDER is the order of the differential equation (length of vector S).  
 ! N\_FUNCTION is the function number (used in FM\_RK14\_F) identifying which function is to be solved.

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USE FMVALS
USE FMZM
IMPLICIT NONE

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INTEGER, PARAMETER :: MAXIMUM_ORDER = 3
INTEGER :: J, K, N_STAGES, N_ORDER, N_FUNCTION
TYPE (FM) :: H, ERROR_EST, S(MAXIMUM_ORDER), S1(MAXIMUM_ORDER), X
TYPE (FM), SAVE :: A(0:34), B(0:34,0:34), C(0:34), FI_J(MAXIMUM_ORDER,0:34), &
    FI(MAXIMUM_ORDER), YK(0:34,MAXIMUM_ORDER), Y(MAXIMUM_ORDER)
EXTERNAL :: FM_RK14_F
INTEGER, SAVE :: COEFF_BASE = 0, COEFF_PRECISION = 0

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CALL FM_ENTER_USER_ROUTINE

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N_STAGES = 35

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IF (COEFF_BASE /= MBASE .OR. COEFF_PRECISION < NDIG) THEN
  COEFF_BASE = MBASE
  COEFF_PRECISION = NDIG
  CALL FM_RK14_COEFFS(A, B, C)
ENDIF

```

! Do the step.

```

YK(0,1:N_ORDER) = S(1:N_ORDER)
DO K = 1, N_STAGES - 1
  YK(K,1:N_ORDER) = 0
  DO J = 0, K-1
    IF (J == K-1) THEN
      Y(1:N_ORDER) = YK(J,1:N_ORDER)
      CALL FM_RK14_F(N_ORDER, N_FUNCTION, X+A(J)*H, Y, FI)
      FI_J(1:N_ORDER,J) = FI(1:N_ORDER)
    ENDIF
    IF (B(K,J) /= 0) THEN
      YK(K,1:N_ORDER) = YK(K,1:N_ORDER) + B(K,J) * FI_J(1:N_ORDER,J)
    ENDIF
  ENDDO
  YK(K,1:N_ORDER) = YK(0,1:N_ORDER) + H * YK(K,1:N_ORDER)
ENDDO

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S1(1:N_ORDER) = 0
DO K = 0, N_STAGES - 1
  IF (K == N_STAGES-1) THEN
    Y(1:N_ORDER) = YK(K,1:N_ORDER)
    CALL FM_RK14_F(N_ORDER, N_FUNCTION, X+A(K)*H, Y, FI)
    S1(1:N_ORDER) = S1(1:N_ORDER) + C(K) * FI(1:N_ORDER)
  ELSE
    S1(1:N_ORDER) = S1(1:N_ORDER) + C(K) * FI_J(1:N_ORDER,K)
  ENDIF
ENDDO

```

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S1(1:N_ORDER) = S(1:N_ORDER) + H * S1(1:N_ORDER)

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ERROR_EST = H * ( NORM2( FI_J(1:N_ORDER,33) - FI_J(1:N_ORDER,1) ) ) / 1000

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CALL FM_EXIT_USER_ROUTINE
END SUBROUTINE FM_RK14_STEP

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SUBROUTINE FM_RK14_COEFFS(A, B, C)

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! Define the coefficients used in the RK14 formula (85 digits).

! These came from:

! [http://www.peterstone.name/Maplepegs/Maple/nmthds/RKcoeff/Runge\\_Kutta\\_schemes/RK14/RKcoeff14a\\_1.pdf](http://www.peterstone.name/Maplepegs/Maple/nmthds/RKcoeff/Runge_Kutta_schemes/RK14/RKcoeff14a_1.pdf)

```

USE FMZM
IMPLICIT NONE

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TYPE (FM) :: A(0:34), B(0:34,0:34), C(0:34)

```

CALL FM\_ENTER\_USER\_ROUTINE

! Many of the coefficients are zero. Initialize A, B, and C here, then  
! skip those definitions below.

A = 0  
B = 0  
C = 0

A( 1) = TO\_FM(' 1 ') / 9  
A( 2) = TO\_FM(' 5 ') / 9  
A( 3) = TO\_FM(' 5 ') / 6  
A( 4) = TO\_FM(' 1 ') / 3  
A( 5) = TO\_FM(' 1 ')  
A( 6) = TO\_FM(' .6699869792727729217646837855059985139388452296384603532851421391683474428303956826239 '  
A( 7) = TO\_FM(' .2970683842138183573895847168082194132233320946989156873791682903324708698499266217383 '  
A( 8) = TO\_FM(' 8 ') / 11  
A( 9) = TO\_FM(' .1401527990421887652761874879669467176298064630825329362873230163439023340348096838456 '  
A(10) = TO\_FM(' .7007010397701507371510998548307493379414070492655464089692218490447945746638665522966 '  
A(11) = TO\_FM(' 4 ') / 11  
A(12) = TO\_FM(' 5 ') / 19  
A(13) = TO\_FM(' .392172246650270859125196642501208648863714315266128052078483e-1 '  
A(14) = TO\_FM(' .8129175029283767629833931592780365061896123726172385507744269795906758195776958783707 '  
A(15) = TO\_FM(' 1 ') / 6  
A(16) = TO\_FM(' 9 ') / 10  
A(17) = TO\_FM(' .6412992574519669233127711938966828094810966516150832254029235721305050295351572963693e-  
A(18) = TO\_FM(' .2041499092834288489277446343010234050271495052413337516288702042649259099754335560687 '  
A(19) = TO\_FM(' .3953503910487605656156713698273243723522272974566594505545766538389345381768585023057 '  
A(20) = TO\_FM(' .6046496089512394343843286301726756276477727025433405494454233461610654618231414976943 '  
A(21) = TO\_FM(' .7958500907165711510722553656989765949728504947586662483711297957350740900245664439313 '  
A(22) = TO\_FM(' .9358700742548033076687228806103317190518903348384916774597076427869494970464842703631 '  
A(23) = TO\_FM(' 1 ') / 6  
A(24) = TO\_FM(' .8129175029283767629833931592780365061896123726172385507744269795906758195776958783707 '  
A(25) = TO\_FM(' .392172246650270859125196642501208648863714315266128052078483e-1 '  
A(26) = TO\_FM(' 4 ') / 11  
A(27) = TO\_FM(' .7007010397701507371510998548307493379414070492655464089692218490447945746638665522966 '  
A(28) = TO\_FM(' .1401527990421887652761874879669467176298064630825329362873230163439023340348096838456 '  
A(29) = TO\_FM(' .2970683842138183573895847168082194132233320946989156873791682903324708698499266217383 '  
A(30) = TO\_FM(' .6699869792727729217646837855059985139388452296384603532851421391683474428303956826239 '  
A(31) = TO\_FM(' 1 ') / 3  
A(32) = TO\_FM(' 5 ') / 9  
A(33) = TO\_FM(' 1 ') / 9  
A(34) = TO\_FM(' 1 ')

C( 0) = TO\_FM(' 1 ') / 56  
C( 1) = TO\_FM(' 3 ') / 512  
C( 2) = TO\_FM(' 3 ') / 256  
C( 4) = TO\_FM(' 9 ') / 512  
C( 6) = TO\_FM(' 3 ') / 128  
C( 7) = TO\_FM(' 15 ') / 512  
C( 9) = TO\_FM(' 9 ') / 256  
C(10) = TO\_FM(' 21 ') / 512  
C(11) = TO\_FM(' 3 ') / 64  
C(13) = TO\_FM(' 27 ') / 512  
C(14) = TO\_FM(' 15 ') / 256  
C(15) = TO\_FM(' 33 ') / 512

C(17) = TO\_FM(' .1053521135717530196914960328878781622276730830805238840416702908213176249782427570033 ' )  
C(18) = TO\_FM(' .1705613462417521823821203385538740858875554878027908047375010369442754416180982144816 ' )  
C(19) = TO\_FM(' .2062293973293519407835264857011048947419142862595424540779715293772640762608018856579 ' )  
C(20) = TO\_FM(' .2062293973293519407835264857011048947419142862595424540779715293772640762608018856579 ' )  
C(21) = TO\_FM(' .1705613462417521823821203385538740858875554878027908047375010369442754416180982144816 ' )  
C(22) = TO\_FM(' .1053521135717530196914960328878781622276730830805238840416702908213176249782427570033 ' )  
C(23) = TO\_FM(' -33 ' ) / 512  
C(24) = TO\_FM(' -15 ' ) / 256  
C(25) = TO\_FM(' -27 ' ) / 512  
C(26) = TO\_FM(' -3 ' ) / 64  
C(27) = TO\_FM(' -21 ' ) / 512  
C(28) = TO\_FM(' -9 ' ) / 256  
C(29) = TO\_FM(' -15 ' ) / 512  
C(30) = TO\_FM(' -3 ' ) / 128  
C(31) = TO\_FM(' -9 ' ) / 512  
C(32) = TO\_FM(' -3 ' ) / 256  
C(33) = TO\_FM(' -3 ' ) / 512  
C(34) = TO\_FM(' 1 ' ) / 56

B( 1, 0) = TO\_FM(' 1 ' ) / 9  
B( 2, 0) = TO\_FM(' -5 ' ) / 6  
B( 2, 1) = TO\_FM(' 25 ' ) / 18  
B( 3, 0) = TO\_FM(' 5 ' ) / 24  
B( 3, 2) = TO\_FM(' 5 ' ) / 8  
B( 4, 0) = TO\_FM(' 29 ' ) / 150  
B( 4, 2) = TO\_FM(' 11 ' ) / 50  
B( 4, 3) = TO\_FM(' -2 ' ) / 25  
B( 5, 0) = TO\_FM(' 1 ' ) / 10  
B( 5, 3) = TO\_FM(' 2 ' ) / 5  
B( 5, 4) = TO\_FM(' 1 ' ) / 2  
B( 6, 0) = TO\_FM(' .103484561636679776672993546511910344499744798201971316606662972828198196507929074598 ' )  
B( 6, 3) = TO\_FM(' .122068887306407222589644082868962077139592714834162134741274656370905593732531152167 ' )  
B( 6, 4) = TO\_FM(' .482574490331246622475134780125688112865919023850168049679401502369641327386232154415 ' )  
B( 6, 5) = TO\_FM(' -.38140960001560699973088624000562020566411307247841147742196992400397674796296698556 ' )  
B( 7, 0) = TO\_FM(' .124380526654094412881516420868799316268491466359671423163289235462806853711761294279 ' )  
B( 7, 4) = TO\_FM(' .226120282197584301422238662979202901196752320742633143965144746028119620664340435602 ' )  
B( 7, 5) = TO\_FM(' .137885887618080880607695837016477814530969417491493385363542870947528858606155278236 ' )  
B( 7, 6) = TO\_FM(' -.67221013399668444974939950741430585695008634152538218285619978253208490386790635967 ' )  
B( 8, 0) = TO\_FM(' .936919065659673815530885456083005933866349695217750085655603386289346442924181510100 ' )  
B( 8, 5) = TO\_FM(' -.61340684345051098722949899564166473562091450712885887100709860683724753553208359970 ' )  
B( 8, 6) = TO\_FM(' .216019825625503063708860097659866573490979433278117320188667670606612864034055761436 ' )  
B( 8, 7) = TO\_FM(' .423695063515761937337619073960976753205867469544123532683115704105552239756119650823 ' )  
B( 9, 0) = TO\_FM(' .838479812409052664616968791372814085980533139224911131069334667010792262519737503487 ' )  
B( 9, 5) = TO\_FM(' -.11794936710097381431975505603129577536796196059073615077761282688752657882487909035 ' )  
B( 9, 6) = TO\_FM(' -.24729902056881265233947383874319459832599284035334013269749842475035010831584129658 ' )  
B( 9, 7) = TO\_FM(' .978080858367729012259313014081291665503740655476733940756599103749962109343737193234 ' )  
B( 9, 8) = TO\_FM(' .217590689243420631360008651767860318344168120024782176879989346706929663046791419792 ' )  
B(10, 0) = TO\_FM(' .615255359769428227954562389614314714333423969064821107453939756921508709933365484409 ' )  
B(10, 5) = TO\_FM(' .592232780324503308042990005798046524738389560444257136834989677308434797282577545500 ' )  
B(10, 6) = TO\_FM(' .470326159963841112217224303205894113455362530746108825010848323660160451665019356813 ' )  
B(10, 7) = TO\_FM(' .29968886384867900085398183709619239913683112167178127918419368588882750409420424246 ' )  
B(10, 8) = TO\_FM(' -.24765687759399491468999227632981082585395806926394709554818863174800909676479057716 ' )  
B(10, 9) = TO\_FM(' .110895029771437682893999851839061714522445173600678718208624598778525250388055024503 ' )  
B(11, 0) = TO\_FM(' .419700073362782579861792864787277787213483656543104611245994538967465542904805771037 ' )  
B(11, 5) = TO\_FM(' -.317987696266205093901912847692712407988609169703103952205634e-2 ' )  
B(11, 6) = TO\_FM(' .806397714906192077260821711520379506393543111567419750119746883965640536777952521350 ' )

B(11, 7) = TO\_FMC ' .975983126412388979093522850684288851314672048003054550357187518555054921329995824199  
B(11, 8) = TO\_FMC ' .778575578158398909027512446452927238999763460594181964958852034513305085047718548920  
B(11, 9) = TO\_FMC ' .204890423831599428189499202098105603312029235081420653574829342040088524274782351662  
B(11,10) = TO\_FMC ' -1.5626157962746818830707094395052782521146289223642436089280537626349225561602972178  
B(12, 0) = TO\_FMC ' .437726782233730163574465242495339811688214967071614123256972922317293974294041673339  
B(12, 8) = TO\_FMC ' .624365027520195208794358628580933625281631216903095917201250460944402824143824858117  
B(12, 9) = TO\_FMC ' .200043097109577314994435165469647856829066232218264969608768069119704887239114382307  
B(12,10) = TO\_FMC ' - .80532836780498303682385716204890291192339288733702931484420589280840750774603025448  
B(12,11) = TO\_FMC ' .211517528067396521915711903523399601316877825157550573051220877040478674306613990587  
B(13, 0) = TO\_FMC ' .283499250363514563095023591920717312247137654896477097768495601239300914306579551378  
B(13, 8) = TO\_FMC ' .249163204855817407538949148805995149459884653585417680098221999507591288576674458719  
B(13, 9) = TO\_FMC ' .230138787854593149638399846373742768772087122638142234223658365573562010865783699395  
B(13,10) = TO\_FMC ' - .32215595669297709872447609246712087818946360476062046104330851071900310989870049382  
B(13,11) = TO\_FMC ' .988442549447664668946335414487885256040819982786014648129299307804937324583961840500  
B(13,12) = TO\_FMC ' - .21301077132888735138430764287592738488663456542957246663209224647221547549855683131  
B(14, 0) = TO\_FMC ' .343511894290243001049432234735147943083353174980701426268650747412312041601045786757  
B(14, 8) = TO\_FMC ' .210451912023627385609097011999010655788807405225626700041905005148763264151801873268  
B(14, 9) = TO\_FMC ' 1.03427452057230411936482926828825709938667999698324740166692913417793163217634902673  
B(14,10) = TO\_FMC ' .600303645864422487051240448206640574939078092406156945467307568641714211716425426287  
B(14,11) = TO\_FMC ' .855938125099619537578012106002407728915062652616416005817268435488127764834196056300  
B(14,12) = TO\_FMC ' - .97723500503676681087226485237252563301310765689283967769744124463491057997058515060  
B(14,13) = TO\_FMC ' - .66002698047929469461622501385632769372057398121997487477755817368796544533227596834  
B(15, 0) = TO\_FMC ' - .14357400167216806953820639993507636665775595437839988097571536728963150444261838822  
B(15, 8) = TO\_FMC ' - .36625327004903997029368579684897479173311908173355220786579136213828240389888077962  
B(15, 9) = TO\_FMC ' .350254975636213681976849406979846524346789082471103574202065474971751829159721055935  
B(15,10) = TO\_FMC ' .360946016362113508931786658758335239823689929864237671889588008396048697054782568349  
B(15,11) = TO\_FMC ' - .26521996755368110635159594683460192364962701245746428486672526069427397871601306826  
B(15,12) = TO\_FMC ' .445699011305698119638911537508839908104336323082226771670762909231511147961495867326  
B(15,13) = TO\_FMC ' .124343093331358243286225595741786448038973408895106741916775999000141977621729255419  
B(15,14) = TO\_FMC ' .413829693239480694403512496204335960426192908674476034447222741881231033308868569827  
B(16, 0) = TO\_FMC ' .356032404425120290975609116398089176264106222379748802653696810150127580505128976082  
B(16, 8) = TO\_FMC ' - .450192758947562595966821779075956175110645100214763601190349 ')  
B(16, 9) = TO\_FMC ' .430527907083710898626656292808782917793030154094709462877146 ')  
B(16,10) = TO\_FMC ' .511973029011022237668556960394071692077125787030651386390680524440504281375541151246  
B(16,11) = TO\_FMC ' .908303638886404260390159124638110213997496214819904630544545254187052815393323608827  
B(16,12) = TO\_FMC ' -1.2392109337193393175737246915153402885441388924860572618608879665100007552209575949  
B(16,13) = TO\_FMC ' - .64904866167176146514167234887906255390540283196719109765440254562354915105598784353  
B(16,14) = TO\_FMC ' .251708904586819292210480529948970541404887852931447491218925635425985377682963093765  
B(16,15) = TO\_FMC ' .779906470345586398810756795282334476023540593411550187020645287929879851319988608557  
B(17, 0) = TO\_FMC ' .130935687406513066406881206418834980127470438213192487844956657556530296569619534119  
B(17,12) = TO\_FMC ' - .93205306798511394590846196276710823785863150968466714212476970175565051738975786101  
B(17,13) = TO\_FMC ' .505374334262299359640090443138590726770942344716122381702745663085652655547883139601  
B(17,14) = TO\_FMC ' .804470341944487979109579109610197797641311868930865361049372199939912941758662925143  
B(17,15) = TO\_FMC ' .59172602949417119052875574277717259844340971924321528178230203407134222992166127834  
B(17,16) = TO\_FMC ' - .40161472215455733706469168490637558773226424795009380467745659930134242948673984557  
B(18, 0) = TO\_FMC ' .207926484466053012541944544000765652167255206144373407979758696985305554917550545773  
B(18,12) = TO\_FMC ' .582695918800085915101902697837284108951406103029871570103106548036064141629810292085  
B(18,13) = TO\_FMC ' - .80170073235881593908334218652585274664055846591963352465549926805065881698632857188  
B(18,14) = TO\_FMC ' .403847643847136940375170821743560570484117290330895506619165536822386238860521369092  
B(18,15) = TO\_FMC ' .854609998055506144225056114567535602510114622033622491802596131021194059200962159560  
B(18,16) = TO\_FMC ' - .20448648093580424270670756969100430790444283755267745623314309891164588146099278914  
B(18,17) = TO\_FMC ' .105328578824431893399799402979093997354240904235172843146582747372367365188241765676  
B(19, 0) = TO\_FMC ' 1.40153449795736021415446247355771306718486452917597733128988131888409635429407909911  
B(19,12) = TO\_FMC ' - .23025200098422126161627241036741562126113029827445562191750101570570311258146692390  
B(19,13) = TO\_FMC ' -7.2110684046691290565958223710687424716585649350996156973248495325768908945066194050  
B(19,14) = TO\_FMC ' .372901560694836335236995327852132340217759566678662388237305709622913736016443541124

B(19,15) = TO\_FMC' -4.7141549572712502067877817939222475701132337322182009801948455220137110350547626648  
B(19,16) = TO\_FMC' -.17636765754534924205384199503279767357490388669560013405931947172361222337991262294  
B(19,17) = TO\_FMC' 7.64130548038698765563029310880237651185173367813937005981851966140144220266574111127  
B(19,18) = TO\_FMC' 3.50602043659751834989896082949744710968212949893375373634159188147070800823352197655  
B(20, 0) = TO\_FMC' 11.9514650694120686799372385830716401674473610826553516824275493462654396835733174209  
B(20,12) = TO\_FMC' 7.79480932108175968783516700231764388220284279598980954919791777616158822520632258045  
B(20,13) = TO\_FMC' -56.450139386732579252356099112090428144046810006134055386359677630112140226291729076  
B(20,14) = TO\_FMC' .912376306930644901344530449290276645709607450403673704749970493658227027495012839891  
B(20,15) = TO\_FMC' -12.733627992543488620194552430919927503816271752991896051684578243737793898281105813  
B(20,16) = TO\_FMC' -.39689592190471971231354281093973667471238307043314787293194118862021186711135161724  
B(20,17) = TO\_FMC' 54.4392141883570886996225765155307791861438378423305334100198542305336689011824705646  
B(20,18) = TO\_FMC' -3.6441163792156923684640699036135064580672147840926673565893423450573740501141560750  
B(20,19) = TO\_FMC' -.80450324991050991089903078795857949931569491321078788074810271839612468949034422587  
B(21, 0) = TO\_FMC' -148.8094265071004884278388682686476255619306120821485965777899951377677370929117632  
B(21,12) = TO\_FMC' -91.729527829125648435793566240232162349522872903635428362913603465786882655388013983  
B(21,13) = TO\_FMC' 707.656144971598359834575719286335716154821128966649562358480474498795767789337915780  
B(21,14) = TO\_FMC' -1.10563611857482440905296961311590930801338308942637769555540 ')  
B(21,15) = TO\_FMC' 176.134591883811372587859898076055660406999516762301686588286912996291141609609787894  
B(21,16) = TO\_FMC' .491384824214880662268898345164454557416884631402764792501960451936899496504529992382  
B(21,17) = TO\_FMC' -684.27800044981494435823753561089508195607716789360027513717997268298218418347912326  
B(21,18) = TO\_FMC' 27.9910604998398258984224332124380407446002518400668686820968895810991697992672738422  
B(21,19) = TO\_FMC' 13.1939710030282333443670964371153238435064159623744975368387222066398949537608733035  
B(21,20) = TO\_FMC' 1.25128781283980445450114974148056006317268830077396406360514134751804098970249919985  
B(22, 0) = TO\_FMC' -9.6730794694819676364412611843321939583995140857187725963492778680680214583036267791  
B(22,12) = TO\_FMC' -4.4699015085850553144384622770196036049783068140875143574880233936706790836330201065  
B(22,13) = TO\_FMC' 45.5127128690952681968241950400052751178905907817398481689041245984012196920096126098  
B(22,14) = TO\_FMC' -.713085086183826912791492024438246129930559805352394367050813e-1 ')  
B(22,15) = TO\_FMC' 11.2273614068412741582590624479939384207826800776794483081522110513351697714459505218  
B(22,16) = TO\_FMC' .126244376717622724516237912909138809361786889819105426371492541686914777310481348245  
B(22,17) = TO\_FMC' -43.543933954948331360581062490724210762381430446762140569378816523593753697654571501  
B(22,18) = TO\_FMC' .787174307543058978398792994996550902064546091443234037811376612477902813309979786716  
B(22,19) = TO\_FMC' .532264696744684215669300708603886690785395776821503853652011892165603372344930229624  
B(22,20) = TO\_FMC' .422422733996325326010225127471388772575086538809603346849794167391050954005095705717  
B(22,21) = TO\_FMC' .859131249503067107308438031499859443441115056294154956398958646615423562116524556319  
B(23, 0) = TO\_FMC' -10.066403244705470240339660690042689147220282475796876527106236043801524494090804448  
B(23, 8) = TO\_FMC' -.36625327004903997029368579684897479173311908173355220786579136213828240389888077962  
B(23, 9) = TO\_FMC' .350254975636213681976849406979846524346789082471103574202065474971751829159721055935  
B(23,10) = TO\_FMC' .360946016362113508931786658758335239823689929864237671889588008396048697054782568349  
B(23,11) = TO\_FMC' -.26521996755368110635159594683460192364962701245746428486672526069427397871601306826  
B(23,12) = TO\_FMC' -6.2708897218146414359055314947887160383935612295739602301940578185331616246743139945  
B(23,13) = TO\_FMC' 48.2079237442562989090702103008195063923492593141636116127889918778040798046242665680  
B(23,14) = TO\_FMC' -.694471689136165640882395180583732834557754169149088630301342e-1 ')  
B(23,15) = TO\_FMC' 12.6810690204850295698341370913609807066108483811412125145427306070793701724650953489  
B(23,16) = TO\_FMC' .119671168968323754838161435501011294100927813964199613229864e-1 ')  
B(23,17) = TO\_FMC' -46.724976499248240800335826824266269559320132165979560704013092633010392633736342305  
B(23,18) = TO\_FMC' 1.33029613326626711314710039298216591399033511191227119235647909906751205113296569734  
B(23,19) = TO\_FMC' 1.00766787503398298353438903619926657771162717793661719905612178795652968013907202793  
B(23,20) = TO\_FMC' .209512051933665091664122388475480702892770753864487241124728406503294010667925100578  
B(23,21) = TO\_FMC' .210134706331264177317735424331396407424412188443757490890226389485516284747891141113  
B(23,22) = TO\_FMC' .952196014417121794175101542454575907376360233658356240546842445184826690518517186553  
B(24, 0) = TO\_FMC' -409.47808167774370877258909740937035762442434160675206834553260358551620237760886998  
B(24, 8) = TO\_FMC' .210451912023627385609097011999010655788807405225626700041905005148763264151801873268  
B(24, 9) = TO\_FMC' 1.03427452057230411936482926828825709938667999698324740166692913417793163217634902673  
B(24,10) = TO\_FMC' .600303645864422487051240448206640574939078092406156945467307568641714211716425426287  
B(24,11) = TO\_FMC' .855938125099619537578012106002407728915062652616416005817268435488127764834196056300  
B(24,12) = TO\_FMC' -250.51699854744786049277765772931613038658405042078207793263939978120268747356142102

B(24,13) = TO\_FMC ' 1946.42466652388427766053750328264758595829850895761427456061026089918613625951401524  
B(24,14) = TO\_FMC ' -3.0450388210231036550610580908686088278695054409760210168421966223178314466054996989  
B(24,15) = TO\_FMC ' 490.626379528281713521208265299168083841598542274061663305100359412876615233718522008  
B(24,16) = TO\_FMC ' 1.56647589531270907115484067013597445739595615245966775319938869084117342471443487192  
B(24,17) = TO\_FMC ' -1881.9742899401117336221726737703587061921590663845305576892756960317929119933570710  
B(24,18) = TO\_FMC ' 75.2592224724847175278837713643303149821620618914245944022930180751661537997299406270  
B(24,19) = TO\_FMC ' 34.5734356980331067622434344736554689696728644793551015800152999093724397634872444844  
B(24,20) = TO\_FMC ' 3.21147679440968961435417361847073755169022966748891627885575411324313568439899341011  
B(24,21) = TO\_FMC ' -.46040804173841439130720140423705884886724509526538282084272965614150792140170744276  
B(24,22) = TO\_FMC ' -.87071833984181052243188413795798624572425204738893657221457481431251621336309441283  
B(24,23) = TO\_FMC ' -7.3935181415830306756701695219552106399918577324913295439263466131938253153940872862  
B(25, 0) = TO\_FMC ' 3.43347475853550878921093496257596781120623891072008458871275578664458303551475269959  
B(25, 8) = TO\_FMC ' .249163204855817407538949148805995149459884653585417680098221999507591288576674458719  
B(25, 9) = TO\_FMC ' .230138787854593149638399846373742768772087122638142234223658365573562010865783699395  
B(25,10) = TO\_FMC ' -.32215595669297709872447609246712087818946360476062046104330851071900310989870049382  
B(25,11) = TO\_FMC ' .988442549447664668946335414487885256040819982786014648129299307804937324583961840500  
B(25,12) = TO\_FMC ' 2.16252799377922507788307841904757354045759225335732709485147995656424695731447613347  
B(25,13) = TO\_FMC ' -16.269986454645742132806564066013948900698755204022885179857750753632327568819704866  
B(25,14) = TO\_FMC ' -.12853450212052455284358341747093501053802903754265450623026518488443528560378848221  
B(25,15) = TO\_FMC ' -8.98915042666504253089307820833379330486511746063552853023189 ')  
B(25,16) = TO\_FMC ' -.3485953632320253338708020185101365019240176725051376496887301361750867676541813193  
B(25,17) = TO\_FMC ' 15.7936194113339807536235187388695574135853387025139737665615827526614052553101160860  
B(25,18) = TO\_FMC ' -.57440333091409506562816548201733582014838366319567547082314583984232559842522810471  
B(25,19) = TO\_FMC ' -.34560203902139329669272249660812498253523722882765530670818338894198985650704675341  
B(25,20) = TO\_FMC ' -.66224149020658509173161999138375778113306799270741868739064504133854458740360013884  
B(25,21) = TO\_FMC ' -.77778812924220416403254645860736430975934720962675911201553677611502731832484417083  
B(25,22) = TO\_FMC ' -.35608419240227491333882723269743736467524081879170658795260634060923363004936073005  
B(25,23) = TO\_FMC ' 4.79282506449930799649797749629840189457296934139359055541771261862435474722265779160  
B(25,24) = TO\_FMC ' .153725464873068577844576387402512082757034273069877432944621 ')  
B(26, 0) = TO\_FMC ' 32.3038520871985442326994734440031535091364975047784629761706142171928114605813985223  
B(26, 5) = TO\_FMC ' -.317987696266205093901912847692712407988609169703103952205634e-2 ')  
B(26, 6) = TO\_FMC ' .806397714906192077260821711520379506393543111567419750119746883965640536777952521350  
B(26, 7) = TO\_FMC ' .975983126412388979093522850684288851314672048003054550357187518555054921329995824199  
B(26, 8) = TO\_FMC ' .778575578158398909027512446452927238999763460594181964958852034513305085047718548920  
B(26, 9) = TO\_FMC ' .204890423831599428189499202098105603312029235081420653574829342040088524274782351662  
B(26,10) = TO\_FMC ' -1.5626157962746818830707094395052782521146289223642436089280537626349225561602972178  
B(26,12) = TO\_FMC ' 16.3429891882310570648504243973927174708753353504154551291766690274419879972597084166  
B(26,13) = TO\_FMC ' -154.54455529354362123073018963147103639931668366960911650170781525495649238820841226  
B(26,14) = TO\_FMC ' 1.56971088703334872692034283417621761466263593582497085965862496468707958908947947188  
B(26,15) = TO\_FMC ' 3.27685545087248131321429817269900731165522404974733600045038526951769313077598588460  
B(26,16) = TO\_FMC ' -.50348924519365317634804072719978362653408109569163239724620427000718631646758189558  
B(26,17) = TO\_FMC ' 153.321151858041665070593767885914694011224363102594556490702148670713911429499613494  
B(26,18) = TO\_FMC ' 7.17568186327720495846766484814784143567826308034865338654018514583315590848812891056  
B(26,19) = TO\_FMC ' -2.9403674867530048194591765989693098921532059438077759794276157404769088650981355956  
B(26,20) = TO\_FMC ' -.66584594607680314447074967602262887028192049319725688787087447830285583694684970322  
B(26,21) = TO\_FMC ' -.46234605499084366122924866856221726117696651401685928423742684491406430687867606188  
B(26,22) = TO\_FMC ' -.20419873358567940153938822861726977884857977482158177767512359106649843522849681001  
B(26,23) = TO\_FMC ' -53.352310643873585051595344116599810797404509049579159779968763906727112391569771034  
B(26,24) = TO\_FMC ' -1.3554871471507865497873218670599640401755450161419132511482067383293601429366562829  
B(26,25) = TO\_FMC ' -1.5719627580123275188290173517145924917768721911444258346186632825709586840386984957  
B(27, 0) = TO\_FMC ' -16.645146748634151287203129440393175876456037113081897820442570161548259239467584758  
B(27, 5) = TO\_FMC ' .592232780324503308042990005798046524738389560444257136834989677308434797282577545500  
B(27, 6) = TO\_FMC ' .470326159963841112217224303205894113455362530746108825010848323660160451665019356813  
B(27, 7) = TO\_FMC ' .29968886384867900085398183709619239913683112167178127918419368588882750409420424246  
B(27, 8) = TO\_FMC ' -.24765687759399491468999227632981082585395806926394709554818863174800909676479057716  
B(27, 9) = TO\_FMC ' .110895029771437682893999851839061714522445173600678718208624598778525250388055024503



B(27,11) = TO\_FMC' -.49171904384622914707066662870419409767808190721067304498886647498364034748888323949  
B(27,12) = TO\_FMC' -11.474315442728949696838949256435253635084245413085317578564839658638985348494168405  
B(27,13) = TO\_FMC' 80.2593166576230272541702485886484400152793366623589987589384940050727853493115840823  
B(27,14) = TO\_FMC' -.38413230398004284762531252675902910374692684134208821920681331074921203482636184660  
B(27,15) = TO\_FMC' 7.28147667468107583471326950926136115767612581862877724348398899410449871401104735520  
B(27,16) = TO\_FMC' -.13269938461224837951057170817603527483682734161675188430181786535262802690654705904  
B(27,17) = TO\_FMC' -81.079983252573072667467928975225524000607071663363298856415623572371668101967605930  
B(27,18) = TO\_FMC' -1.2503749283562063952176818565617911996225374749240318631924346294018197298688520905  
B(27,19) = TO\_FMC' 2.59263594969543681023776379504377324994226447359296888083758688356006843434981849191  
B(27,20) = TO\_FMC' -.30144029834640453983016399726052687526443153727564149534207970744575525861374881107  
B(27,21) = TO\_FMC' .221384460789832337451706451572773791695246839057318414317957361770432316698526521736  
B(27,22) = TO\_FMC' .827577274771892931955989870974693152996276435429809890549707872973435398089631530569  
B(27,23) = TO\_FMC' 18.9960662040611520464672450037243263998175161412237158936671867499994356976969694352  
B(27,24) = TO\_FMC' .26923194640963968562346801512833416746005191034891284512118666889106686145776773566  
B(27,25) = TO\_FMC' 1.62674827447066537462989364929628933988125029284183680279020143050484769780352863639  
B(27,26) = TO\_FMC' .491719043846229147070666628704194097678081907210673044988866474983640347488883239492  
B(28, 0) = TO\_FMC' .838479812409052664616968791372814085980533139224911131069334667010792262519737503487  
B(28, 5) = TO\_FMC' -.11794936710097381431975505603129577536796196059073615077761282688752657882487909035  
B(28, 6) = TO\_FMC' -.24729902056881265233947383874319459832599284035334013269749842475035010831584129658  
B(28, 7) = TO\_FMC' .978080858367729012259313014081291665503740655476733940756599103749962109343737193234  
B(28, 8) = TO\_FMC' .217590689243420631360008651767860318344168120024782176879989346706929663046791419792  
B(28,10) = TO\_FMC' .137585606763325224865659632196787746647447222975084865975440090398783377163957572786  
B(28,11) = TO\_FMC' .439870229715046685058790092341545026046103890294261359042580883994320563544728474507  
B(28,13) = TO\_FMC' -.51370081376819334195700445661863030373875736364196403008697121699333983059059313434  
B(28,14) = TO\_FMC' .826355691151315508644211308399153458701423158616168576922419497747188233542014118321  
B(28,15) = TO\_FMC' 25.7018139719811832625873882972519939511136556341960078182470273709164512916981313440  
B(28,23) = TO\_FMC' -25.701813971981183262587388297251993951113655634196007818247027370916451291698131344  
B(28,24) = TO\_FMC' -.82635569115131550864421130839915345870142315861616857692241949774718823354201411832  
B(28,25) = TO\_FMC' .513700813768193341957004456618630303738757363641964030086971216993339830590593134346  
B(28,26) = TO\_FMC' -.43987022971504668505879009234154502604610389029426135904258088399432056354472847450  
B(28,27) = TO\_FMC' -.13758560676332522486565963219678774664744722297508486597544009039878337716395757278  
B(29, 0) = TO\_FMC' .124380526654094412881516420868799316268491466359671423163289235462806853711761294279  
B(29, 4) = TO\_FMC' .226120282197584301422238662979202901196752320742633143965144746028119620664340435602  
B(29, 5) = TO\_FMC' .137885887618080880607695837016477814530969417491493385363542870947528858606155278236  
B(29, 6) = TO\_FMC' -.67221013399668444974939950741430585695008634152538218285619978253208490386790635967  
B(29, 9) = TO\_FMC' -.85623897508542835475534976987950177211212159741156380285506653858506127410402252229  
B(29,10) = TO\_FMC' -1.9633752286685890892826285002809381398818044051826740455357566315269169500833538451  
B(29,11) = TO\_FMC' -.23233282272411940123724625730892184725010819923041999497821803199052620457188722596  
B(29,13) = TO\_FMC' 4.30660719086453349461668936876562947772432562053478092626764039360850075857010049587  
B(29,14) = TO\_FMC' -2.9272296324946548265978791120239044668768739495063361263077866352629923674849987865  
B(29,15) = TO\_FMC' -82.313166639785894445449233410545870773576196642813868939506013093564171819486459970  
B(29,23) = TO\_FMC' 82.3131666397858944454492334105458707735761966428138689395060130935641718194864599704  
B(29,24) = TO\_FMC' 2.92722963249465482659787911202390446687687394950633612630778663526299236748499878651  
B(29,25) = TO\_FMC' -4.3066071908645334946166893687656294777243256205347809262676403936085007585701004958  
B(29,26) = TO\_FMC' .232332822724119401237246257308921847250108199230419994978218031990526204571887225960  
B(29,27) = TO\_FMC' 1.96337522866858908928262850028093813988180440518267404553575663152691695008335384516  
B(29,28) = TO\_FMC' .856238975085428354755349769879501772112121597411563802855066538585061274104022522297  
B(30, 0) = TO\_FMC' .103484561636679776672993546511910344499744798201971316606662972828198196507929074598  
B(30, 3) = TO\_FMC' .122068887306407222589644082868962077139592714834162134741274656370905593732531152167  
B(30, 4) = TO\_FMC' .482574490331246622475134780125688112865919023850168049679401502369641327386232154415  
B(30, 5) = TO\_FMC' -.38140960001560699973088624000562020566411307247841147742196992400397674796296698556  
B(30, 7) = TO\_FMC' -.55049952531080232413838850702050817741141431100003756171283632064244734987451410659  
B(30, 9) = TO\_FMC' -.71191581158518922788764826204379438757829188240674557049576521397105747998786301638  
B(30,10) = TO\_FMC' -.58412960567155134043298873015848087209533532964522759570705244100654176766834630091  
B(30,13) = TO\_FMC' 2.11046308125864932128717300046622750300375054278936987850718228771088147061894331874  
B(30,14) = TO\_FMC' -.83749473673957213552574202300103799269526017533512351774055292983345327937414631628

B(30,15) = TO\_FMC' 5.10021499072320914075295969043344113107545060862804249159734638844513541296521716555  
B(30,23) = TO\_FMC' -5.10021499072320914075295969043344113107545060862804249159734638844513541296521716555  
B(30,24) = TO\_FMC' .837494736739572135525742023001037992695260175335123517740552929833453279374146316284  
B(30,25) = TO\_FMC' -2.1104630812586493212871730004662275030037505427893698785071822877108814706189433187  
B(30,27) = TO\_FMC' .584129605671551340432988730158480872095335329645227595707052441006541767668346300910  
B(30,28) = TO\_FMC' .711915811585189227887648262043794387578291882406745570495765213971057479987863016385  
B(30,29) = TO\_FMC' .550499525310802324138388507020508177411414311000037561712836320642447349874514106596  
B(31, 0) = TO\_FMC' 29 ') / 150  
B(31, 2) = TO\_FMC' 11 ') / 50  
B(31, 3) = TO\_FMC' -2 ') / 25  
B(31, 6) = TO\_FMC' .109993425580724703919462404865068340845119058295846426463652427145968754999400265475  
B(31, 7) = TO\_FMC' -.25429704807627016138406850699715312214183562697670392084624216561641798752690429824  
B(31, 9) = TO\_FMC' .865570777116694254343770343821098281832847401233011859346736813276251089205124275931  
B(31,10) = TO\_FMC' 3.32416449114093083106799552786572018336860092936986407160199838603992063578140986504  
B(31,13) = TO\_FMC' -12.010222331597793388235238514866184126030194263399681512727695284620350021102167281  
B(31,14) = TO\_FMC' .476601466242493239430442776862061899602963782003580209482572024269431555119657612550  
B(31,15) = TO\_FMC' -29.024301122103639052580262321365409959625122133247091069152398706019164507085467440  
B(31,23) = TO\_FMC' 29.0243011221036390525802623213654099596251221332470910691523987060191645070854674407  
B(31,24) = TO\_FMC' -.47660146624249323943044277686206189960296378200358020948257202426943155511965761255  
B(31,25) = TO\_FMC' 12.0102223315977933882352385148661841260301942633996815127276952846203500211021672810  
B(31,27) = TO\_FMC' -3.3241644911409308310679955278657201833686009293698640716019983860399206357814098650  
B(31,28) = TO\_FMC' -.86557077711669425434377034382109828183284740123301185934673681327625108920512427593  
B(31,29) = TO\_FMC' .254297048076270161384068506997153122141835626976703920846242165616417987526904298244  
B(31,30) = TO\_FMC' -.10999342558072470391946240486506834084511905829584642646365242714596875499940026547  
B(32, 0) = TO\_FMC' -5 ') / 6  
B(32, 1) = TO\_FMC' 25 ') / 18  
B(32, 4) = TO\_FMC' -3 ') / 4  
B(32, 6) = TO\_FMC' -.49252954371802630442268204911402132020021468158065778471907408396443463700487493425  
B(32,30) = TO\_FMC' .492529543718026304422682049114021320200214681580657784719074083964434637004874934256  
B(32,31) = TO\_FMC' 3 ') / 4  
B(33, 0) = TO\_FMC' 1 ') / 9  
B(33, 2) = TO\_FMC' -2 ') / 9  
B(33,32) = TO\_FMC' 2 ') / 9  
B(34, 0) = TO\_FMC' .285835140388971558796088842163836414852927537894596466892432289755349015255979226202  
B(34, 1) = TO\_FMC' 7 ') / 24  
B(34, 2) = TO\_FMC' 7 ') / 32  
B(34, 4) = TO\_FMC' 21 ') / 128  
B(34, 6) = TO\_FMC' .218194354945556658327188241581352107093288824322187941141516432711696743953191127277  
B(34, 7) = TO\_FMC' .180392898478697766863635221946775437719620053641849228562434721051416375970367952718  
B(34, 9) = TO\_FMC' .205713839404845018859120755122929542277570094982808905393991478938622850494280484398  
B(34,10) = TO\_FMC' .242715791581770239970282927959446515762745971386670541948576352285954919662591397840  
B(34,11) = TO\_FMC' .246465780813629305833609291181891407799228103869305705137021013528421337979041793074  
B(34,12) = TO\_FMC' -3.4499194079089082497983415460162266206037046061493164428832655233811284525249892789  
B(34,13) = TO\_FMC' .228875562160036081760729060738458584294220372552740218459294839251128133427861795995  
B(34,14) = TO\_FMC' .283290599702151415321527419056733335978436595493855789831404842659507070842418206606  
B(34,15) = TO\_FMC' 3.21085125837766640960131490544236787005557320332238709851298499988057712000817312328  
B(34,16) = TO\_FMC' -.22353877736484569992023375621416250796412523008367403208990162754458983951773735824  
B(34,17) = TO\_FMC' -.70712115720441907351872728620748721213009123195520616079105219285712476121117959341  
B(34,18) = TO\_FMC' 3.21123345150287080408174729202856500893260034443022374324958803415719588559022889362  
B(34,19) = TO\_FMC' 1.40954348309669766030414474301123175769045945573548986357321875282117831097819965796  
B(34,20) = TO\_FMC' -.15136205344374261312160227674251811109096302620367605594945903537126676489247541812  
B(34,21) = TO\_FMC' .372350574527014276454724080214619984397121028202148298737356824383668332379812146564  
B(34,22) = TO\_FMC' .252978746406361336722199907762141285915775728129414319261082478036718273942161724369  
B(34,23) = TO\_FMC' -3.2108512583776664096013149054423678700555732033223870985129849998805771200081731232  
B(34,24) = TO\_FMC' -.28329059970215141532152741905673333597843659549385578983140484265950707084241820660  
B(34,25) = TO\_FMC' -.22887556216003608176072906073845858429422037255274021845929483925112813342786179599

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B(34,26) = TO_FM(' -.24646578081362930583360929118189140779922810386930570513702101352842133797904179307  
B(34,27) = TO_FM(' -.24271579158177023997028292795944651576274597138667054194857635228595491966259139784  
B(34,28) = TO_FM(' -.20571383940484501885912075512292954227757009498280890539399147893862285049428048439  
B(34,29) = TO_FM(' -.18039289847869776686363522194677543771962005364184922856243472105141637597036795271  
B(34,30) = TO_FM(' -.21819435494555665832718824158135210709328882432218794114151643271169674395319112727  
B(34,31) = TO_FM(' -21 ') / 128  
B(34,32) = TO_FM(' -7 ') / 32  
B(34,33) = TO_FM(' -7 ') / 24
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CALL FM_EXIT_USER_ROUTINE  
END SUBROUTINE FM_RK14_COEFFS
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