

Example 1. Compute $f(x) = x^2 - x + 3$ using different formulas.

X is the interval [-0.5 , 1.0]

x^2 gives [0.000 , 1.000]

$x \cdot x$ gives [-0.500 , 1.000]

$x^2 - x + 3$ gives [2.000 , 4.500]. Magnification = 2.500

$x \cdot x - x + 3$ gives [1.500 , 4.500]. Magnification = 3.000

$x \cdot (x - 1) + 3$ gives [1.500 , 3.750]. Magnification = 2.250

$(x - 0.5)^2 + 2.75$ gives [2.750 , 3.750]. Magnification = 1.000

X is the interval [0.1 , 1.0]

$x^2 - x + 3$ gives [2.010 , 3.900]. Magnification = 7.560

$x \cdot x - x + 3$ gives [2.010 , 3.900]. Magnification = 7.560

$x \cdot (x - 1) + 3$ gives [2.100 , 3.000]. Magnification = 3.600

$(x - 0.5)^2 + 2.75$ gives [2.750 , 3.000]. Magnification = 1.000

X is the interval [0.9 , 1.0]

$x^2 - x + 3$ gives [2.810 , 3.100]. Magnification = 3.222

$x \cdot x - x + 3$ gives [2.810 , 3.100]. Magnification = 3.222

$x \cdot (x - 1) + 3$ gives [2.900 , 3.000]. Magnification = 1.111

$(x - 0.5)^2 + 2.75$ gives [2.910 , 3.000]. Magnification = 1.000

X is the interval [0.99 , 1.0]

$X^2 - X + 3$ gives [2.980 , 3.010]. Magnification = 3.020

$X^2 - X + 3$ gives [2.980 , 3.010]. Magnification = 3.020

$X(X - 1) + 3$ gives [2.990 , 3.000]. Magnification = 1.010

$(X - 0.5)^2 + 2.75$ gives [2.990 , 3.000]. Magnification = 1.000

Example 2. Sum $1 / n^7$ from 1 to 100,000 =

1.0083492773819228268397975498496300979331385605652M+0

1.0083492773819228268397975498496300979331385605653M+0

The two endpoints agree to about 51 decimal digits.

Example 3. Integrate $\sin(t) / t$ from $t = 0$ to $t = 20$:

1.5482417010434398401636433421295136922615733621093M+0

1.5482417010434398401636433421295136922615733621094M+0

The two endpoints agree to about 54 decimal digits.

Example 4. Step around the unit circle multiple times using a recurrence

- 1 trips. The two endpoints for x agree to about 54 decimal digits.
- 2 trips. The two endpoints for x agree to about 52 decimal digits.
- 3 trips. The two endpoints for x agree to about 49 decimal digits.
- 4 trips. The two endpoints for x agree to about 47 decimal digits.
- 5 trips. The two endpoints for x agree to about 44 decimal digits.
- 6 trips. The two endpoints for x agree to about 42 decimal digits.
- 7 trips. The two endpoints for x agree to about 39 decimal digits.
- 8 trips. The two endpoints for x agree to about 36 decimal digits.
- 9 trips. The two endpoints for x agree to about 34 decimal digits.
- 10 trips. The two endpoints for x agree to about 31 decimal digits.
- 11 trips. The two endpoints for x agree to about 29 decimal digits.
- 12 trips. The two endpoints for x agree to about 26 decimal digits.
- 13 trips. The two endpoints for x agree to about 24 decimal digits.
- 14 trips. The two endpoints for x agree to about 21 decimal digits.
- 15 trips. The two endpoints for x agree to about 18 decimal digits.
- 16 trips. The two endpoints for x agree to about 16 decimal digits.
- 17 trips. The two endpoints for x agree to about 13 decimal digits.
- 18 trips. The two endpoints for x agree to about 11 decimal digits.
- 19 trips. The two endpoints for x agree to about 8 decimal digits.
- 20 trips. The two endpoints for x agree to about 6 decimal digits.
- 21 trips. The two endpoints for x agree to about 3 decimal digits.
- 22 trips. The two endpoints for x agree to about 1 decimal digits.
- 23 trips. The two endpoints for x agree to about 0 decimal digits.

X =

-5.8564010117471012542822698126300121646708464338966M+1

6.0564010117471012542822698126300121646708464338966M+1

Y =

-5.9564010117471012542822698126300121646708464338966M+1

5.9564010117471012542822698126300121646708464338966M+1

45 trips. The two endpoints for x agree to about 38 decimal digits.
 50 trips. The two endpoints for x agree to about 36 decimal digits.
 55 trips. The two endpoints for x agree to about 33 decimal digits.
 60 trips. The two endpoints for x agree to about 30 decimal digits.
 65 trips. The two endpoints for x agree to about 28 decimal digits.
 70 trips. The two endpoints for x agree to about 25 decimal digits.
 75 trips. The two endpoints for x agree to about 23 decimal digits.
 80 trips. The two endpoints for x agree to about 20 decimal digits.
 85 trips. The two endpoints for x agree to about 18 decimal digits.
 90 trips. The two endpoints for x agree to about 15 decimal digits.
 95 trips. The two endpoints for x agree to about 12 decimal digits.
 100 trips. The two endpoints for x agree to about 10 decimal digits.
 105 trips. The two endpoints for x agree to about 7 decimal digits.
 110 trips. The two endpoints for x agree to about 5 decimal digits.
 115 trips. The two endpoints for x agree to about 2 decimal digits.

X =

8.7369369964583434983069574632632545487869869160581M-1
 8.7891966044189282478553606151779971191942978535848M-1

Y =

4.7914069370368603750977134527639252500817646327574M-1
 4.8436665449974451246461166046786678204890755702842M-1

The two endpoints for x agree to about 2 decimal digits.

Example 5. $2 * \text{Product } 4^n / (4^n - 1)$ from 1 to 10,000. =

3.1415141186819220469785580507138775513425133394700M+0
 3.1415141186819220469785580507138775513425133394701M+0

The two endpoints of $y(30)$ agree to about 52 decimal digits.

Example 6. Differential equation. $y' = -y' / 10 - 2 * y / (x+2)$

X =	3.0000000000	V =	0.831136520960373	-0.452290751829937	Endpoints of V(1) agree to	53 digits
X =	6.0000000000	V =	-0.765883193156779	-0.352158220002235	Endpoints of V(1) agree to	52 digits
X =	9.0000000000	V =	-0.821640364088937	0.257782946044846	Endpoints of V(1) agree to	52 digits
X =	12.0000000000	V =	0.193374374107742	0.325345242516926	Endpoints of V(1) agree to	51 digits
X =	15.0000000000	V =	0.786966466722528	0.052221392312944	Endpoints of V(1) agree to	51 digits
X =	18.0000000000	V =	0.572088550349089	-0.166039347343631	Endpoints of V(1) agree to	50 digits
X =	21.0000000000	V =	-0.008218454426127	-0.190332029943315	Endpoints of V(1) agree to	48 digits
X =	24.0000000000	V =	-0.440713401930602	-0.086656234642814	Endpoints of V(1) agree to	49 digits
X =	27.0000000000	V =	-0.515393780043246	0.031589999652254	Endpoints of V(1) agree to	48 digits
X =	30.0000000000	V =	-0.307766881676032	0.095343042626880	Endpoints of V(1) agree to	48 digits

$y(30) =$

-3.0776688167603169203382987651096461581717811055464M-1
 -3.0776688167603169203382987651096461581717811055418M-1

$y'(30) =$

9.5343042626880286284843246841489833596602442626286M-2
 9.5343042626880286284843246841489833596602442626426M-2

The two endpoints of $y(30)$ agree to about 48 decimal digits.

Example 7. Differential equation. $y'' = -y' / 10 - 200 * y / (x+2)$

X = 3.0000000000	V = -0.102742064000739	-0.215544901796916	Endpoints of V(1) agree to 47 digits
X = 6.0000000000	V = 0.078485157239314	-0.348572728869748	Endpoints of V(1) agree to 39 digits
X = 9.0000000000	V = -0.037481204238851	-0.383488535010064	Endpoints of V(1) agree to 33 digits
X = 12.0000000000	V = 0.013531679745067	-0.333961311141985	Endpoints of V(1) agree to 27 digits
X = 15.0000000000	V = 0.075496648457680	0.094822963645320	Endpoints of V(1) agree to 23 digits
X = 18.0000000000	V = -0.072047742909790	0.022294258042798	Endpoints of V(1) agree to 19 digits
X = 21.0000000000	V = 0.063355820821447	0.032461966986868	Endpoints of V(1) agree to 14 digits
X = 24.0000000000	V = -0.029168578777506	-0.135277237857925	Endpoints of V(1) agree to 10 digits
X = 27.0000000000	V = -0.036220481301623	0.094265735252438	Endpoints of V(1) agree to 7 digits
X = 30.0000000000	V = 0.025275350395373	0.090890872993315	Endpoints of V(1) agree to 3 digits

$y(30) =$

2.5268274663776318683290452013354524008325842304305M-2
2.5282426126969610817979865312140233217038151226782M-2

$y'(30) =$

9.0872770822409973135783151703562888452619638318962M-2
9.0908975164219866493786176735651583524503965546846M-2

The two endpoints of $y(30)$ agree to about 3 decimal digits.

Example 8. Solve a "random" NxN linear system.

For N = 10 the solution elements agree to between 51 and 53 significant digits.
For N = 20 the solution elements agree to between 47 and 50 significant digits.
For N = 30 the solution elements agree to between 44 and 48 significant digits.
For N = 40 the solution elements agree to between 36 and 46 significant digits.
For N = 50 the solution elements agree to between 35 and 43 significant digits.
For N = 60 the solution elements agree to between 28 and 40 significant digits.
For N = 70 the solution elements agree to between 27 and 37 significant digits.
For N = 80 the solution elements agree to between 23 and 34 significant digits.
For N = 90 the solution elements agree to between 16 and 33 significant digits.
For N = 100 the solution elements agree to between 16 and 30 significant digits.

Example 9. Newton's method starting with a single point.

Iteration 1. $X =$
8.6787944117144232159552377016146086744581113103176M-1
8.6787944117144232159552377016146086744581113103177M-1

Iteration 2. $X =$
8.5278337341640992137798507086554877127484032504254M-1
8.5278337341640992137798507086554877127484032504255M-1

Iteration 3. $X =$
8.5260552636892205445692227465314123881418241936054M-1
8.5260552636892205445692227465314123881418241936055M-1

Iteration 4. $X =$
8.5260550201372594802652409548836209726836629454221M-1
8.5260550201372594802652409548836209726836629454222M-1

Iteration 5. $X =$
8.5260550201372549134647241469547803262670602201255M-1
8.5260550201372549134647241469547803262670602201256M-1

Iteration 6. $X =$
8.5260550201372549134647241469531746689845330015140M-1
8.5260550201372549134647241469531746689845330015141M-1

Iteration 7. $X =$
8.5260550201372549134647241469531746689845330015140M-1
8.5260550201372549134647241469531746689845330015141M-1

Iteration 8. $X =$
8.5260550201372549134647241469531746689845330015140M-1
8.5260550201372549134647241469531746689845330015141M-1

Iteration 9. $X =$
8.5260550201372549134647241469531746689845330015140M-1
8.5260550201372549134647241469531746689845330015141M-1

Iteration 10. $X =$
8.5260550201372549134647241469531746689845330015140M-1
8.5260550201372549134647241469531746689845330015141M-1

Example 10. Newton's method starting with an interval containing a root

Iteration 1. $X =$
8.2582914751966549600358492004459062470952152212196M-1
9.1669483793964048943502242929035570875441708134219M-1

Iteration 2. $X =$
8.5153653288747945014206990942716504355878954736541M-1
8.5410378857010420828800408165620227158607956580201M-1

Iteration 3. $X =$
8.5260511282637639422279677005694780814425748090707M-1
8.5260596120050999165916620170701727472347232648131M-1

Iteration 4. X =

8.5260550201370357420197023626581603375273929231293M-1

8.5260550201374929467463703975692934764436548299921M-1

Iteration 5. X =

8.5260550201372549134647241466280563503694077596793M-1

8.5260550201372549134647241472919881356741988716405M-1

Iteration 6. X =

8.5260550201372549134647241469531746689845330015140M-1

8.5260550201372549134647241469531746689845330015141M-1

Iteration 7. X =

8.5260550201372549134647241469531746689845330015140M-1

8.5260550201372549134647241469531746689845330015141M-1

Iteration 8. X =

8.5260550201372549134647241469531746689845330015140M-1

8.5260550201372549134647241469531746689845330015141M-1

Iteration 9. X =

8.5260550201372549134647241469531746689845330015140M-1

8.5260550201372549134647241469531746689845330015141M-1