MODULE FMZM_1
USE FMVALS, ONLY : MULTI, FM, IM, ZM

! FMZM 1.4                           David M. Smith
!
! This module extends the definition of the basic Fortran arithmetic and function operations so
! they also apply to multiple precision numbers, using version 1.3 of FM.
! There are three multiple precision data types:
!   FM  (multiple precision real)
!   IM  (multiple precision integer)
!   ZM  (multiple precision complex)
!
! For some examples and general advice about using these multiple-precision data types, see the
! program SampleFM.f95.
!
! Most of the functions defined in this module are multiple precision versions of standard Fortran
! functions. In addition, there are functions for direct conversion, formatting, and some
! mathematical special functions.
!
! TO_FM is a function for converting other types of numbers to type FM. Note that TO_FM(3.12)
! converts the REAL constant to FM, but it is accurate only to single precision, since the number
! 3.12 cannot be represented exactly in binary and has already been rounded to single precision.
! Similarly, TO_FM(3.12D0) agrees with 3.12 to double precision accuracy, and TO_FM('3.12') or
! TO_FM(312)/TO_FM(100) agrees to full FM accuracy.
!
! TO_IM converts to type IM, and TO_ZM converts to type ZM.
!
! Functions are also supplied for converting the three multiple precision types to the other
! numeric data types:
!   TO_INT  converts to machine precision integer
!   TO_SP   converts to single precision
!   TO_DP   converts to double precision
!   TO_SPZ  converts to single precision complex
!   TO_DPZ  converts to double precision complex
!
! WARNING: When multiple precision type declarations are inserted in an existing program, take
! care in converting functions like DBLE(X), where X has been declared as a multiple
! precision type. If X was single precision in the original program, then replacing
! the DBLE(X) by TO_DP(X) in the new version could lose accuracy. For this reason, the
! Fortran type-conversion functions defined in this module assume that results should
! be multiple precision whenever inputs are. Examples:
!   DBLE(TO_FM('1.23E+123456'))  is type FM
!   REAL(TO_FM('1.23E+123456'))  is type FM
!   REAL(TO_ZM('3.12+4.56i'))    is type FM  = TO_FM('3.12')
!   INT(TO_FM('1.23'))           is type IM  = TO_IM(1)
!   INT(TO_IM('1E+23'))          is type IM
!   CMPLX(TO_FM('1.23'),TO_FM('4.56')) is type ZM
!
! IS_OVERFLOW, IS_UNDERFLOW, and IS_UNKNOWN are logical functions for checking whether a multiple
! precision number is in one of the exception categories. Testing to see if a type FM number is
! in the +overflow category by directly using an IF can be tricky. When MAFM is +overflow, the
! statement
!       IF (MAFM == TO_FM(' +OVERFLOW ')) THEN
! will return false, since the comparison routine cannot be sure that two different overflowed
! results would have been equal if the overflow threshold had been higher. Instead, use
!       IF (IS_OVERFLOW(MAFM)) THEN
which will be true if MAFM is + or - overflow.

Programs using this module may sometimes need to call FM, IM, or ZM routines directly. This is normally the case when routines are needed that are not Fortran intrinsics, such as the formatting subroutine FM_FORM. In a program using this module, suppose MAFM has been declared with TYPE (FM) :: MAFM. To convert the number to a character string with F65.60 format, use

```
CALL FM_FORM('F65.60',MAFM,ST1)
```

WARNING: To be safe, all multiple precision variables in a user's program should be declared as type (FM), (IM), or (ZM), and any direct calls to subroutines should be the kind with the underscore. Subroutines that define one or more multiple precision output values, such as computing pi using

```
CALL FM_PI(PI)
```

automatically cause PI to be put into the FM saved variable area of storage. Calling the low-level routine (CALL FM_PI(MFM)) would cause PI to be treated as an FM temporary variable if PI had not been previously defined in the program. Then the value of PI could be discarded before the program is finished using it.

In subroutine or function subprograms all multiple precision variables that are local to that routine should be declared with the SAVE attribute. It is not an error to omit SAVE, but if the compiler creates new copies of the variables for each call to the routine, then the program will leak memory.

Type (FM), (IM), or (ZM) variables cannot have their multiple precision values initialized in the declaration statement, as can ordinary variables. If the original program had

```
DOUBLE PRECISION :: X = 2.3D0
```

then the corresponding FM version would have

```
TYPE (FM), SAVE :: X
... (other declarations) ...
X = TO_FM( '2.3' )
```

An attempt to use a multiple precision variable that has not been defined will be detected by the routines in this module and an error message printed.

For each of the operations =, ==, /=, <, <=, >, >=, +, -, *, /, and **, the interface module defines all mixed mode variations involving one of the three multiple precision derived types and another argument having one of the types: \{ integer, real, double, complex, complex double, FM, IM, ZM \}. So mixed mode expressions such as

```
MAFM = 12
MAFM = MAFM + 1
IF (ABS(MAFM) > 1.0D-23) THEN
```

are handled correctly.

Not all the named functions are defined for all three multiple precision derived types, so the list below shows which can be used. The labels "real", "integer", and "complex" refer to types FM, IM, and ZM respectively, "string" means the function accepts character strings (e.g., TO_FM('3.45')), and "other" means the function can accept any of the machine precision data types integer, real, double, complex, or complex double. For functions that accept two or more arguments, like ATAN2 or MAX, all the arguments must be of the same type.

Note that TO_ZM also has a 2-argument form: TO_ZM(2,3) for getting 2 + 3*i. CMPLX can be used for that, as in CMPLX( TO_FM(2) , TO_FM(3) ), but the 2-argument form is more concise. The 2-argument form is available for machine precision integer, single and double precision real pairs. For others, such as X and Y being type(fm), just use CMPLX(X,Y).

Fortran's 2-argument version of atan(x,y) is also provided. It is the same as the older atan2. Functions in this list that are not provided by standard Fortran, such as special functions, have more information about their arguments farther down.
! AVAILABLE FUNCTIONS:

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! GCD                           integer
! HUGE                          real  integer  complex
! HYPOT                         real
! INCOMPLETE_BETA               real
! INCOMPLETE_GAMMA1             real
! INCOMPLETE_GAMMA2             real
! INT                           real  integer  complex
! LOG                           real  complex
! LOG10                          real
! LOG_ERFC                       real
! LOG_GAMMA                      real  complex
! LOG_INTEGRAL                   real
! MAX                           real  integer
! MAXEXponent                    real
! MIN                            real  integer
! MINExponent                    real
! MOD                           real  integer
! MODulo                         real  integer
! MULTIPLY_MOD                   integer
! NEAREST                        real
! NINT                           real  integer  complex
! NORM2                          real
! POCHHAMMER                     real
! POLYgamma                      real
! POWER_MOD                      integer
! PRECISION                      real  complex
! PSI                            real
! RADIX                          real  integer  complex
! RANGE                          real  integer  complex
! REAL                           real  integer  complex
! RRSPACING                      real
! SCALE                          real  complex
! SETExponent                    real
! SIGN                           real  integer
! SIN                            real  complex
! SINH                           real  complex
! SIN_INTEGRAL                   real
! SINH_INTEGRAL                  real
! SPACING                        real
! SQRT                           real  complex
! TAN                            real  complex
! TANH                           real  complex
! TINY                           real  integer  complex
! TO_FM                           real  integer  complex  string  other
! TO_TM                           real  integer  complex  string  other
! TO_ZM                           real  integer  complex  string  other
! TO_INT                          real  integer  complex
! TO_SP                           real  integer  complex
! TO_DP                           real  integer  complex
! TO_SPZ                          real  integer  complex
! TO_DPZ                          real  integer  complex
! IS_OVERFLOW                     real  integer  complex
! IS_UNDERFLOW                    real  integer  complex
! IS_UNKNOWN                      real  integer  complex

! SUBROUTINES THAT DO NOT CORRESPOND TO ANY FUNCTION ABOVE:
1. Type (FM). MA, MB, MC refer to type (FM) numbers.

  FM_COSH_SINH(MA, MB, MC)  \( MB = \text{COSH}(MA), \ MC = \text{SINH}(MA) \)
  Faster than making two separate calls.

  FM_COS_SIN(MA, MB, MC)  \( MB = \text{COS}(MA), \ MC = \text{SIN}(MA) \)
  Faster than making two separate calls.

  FM_EULER(MA)  \( MA = \text{Euler's constant} \ (0.5772156649...) \)

  FM_FLAG(K)  \( K = K\text{FLAG} \) get the value of the FM condition flag -- stored in
  the internal FM variable KFLAG in module FMVALS.

  FM_FORM(FORM, MA, STRING)  MA is converted to a character string using format FORM and
  returned in STRING. FORM can represent I, F, E, or ES formats.
  Example:
  CALL FMFORM('F60.40', MA, STRING)

  FM_FPRINT(FORM, MA)  Print MA on unit KW using FORM format.

  FM_PI(MA)  \( MA = \pi \)

  FM_PRINT(MA)  Print MA on unit KW using current format.

  FM_RANDOM_NUMBER(X)  X is returned as a double precision random number, uniformly
  distributed on the open interval (0,1). It is a high-quality, long-period generator based on
  49-digit prime numbers. Note that X is double precision, unlike the similar Fortran
  intrinsic random number routine, which can return a single
  or double precision result.
  A default initial seed is used if FM_RANDOM_NUMBER is called
  without calling FM_RANDOM_SEED_PUT first.

  FM_RANDOM_SEED_GET(SEED)  returns the seven integers SEED(1) through SEED(7) as the current
  seed for the FM_RANDOM_NUMBER generator.

  FM_RANDOM_SEED_PUT(SEED)  initializes the FM_RANDOM_NUMBER generator using the seven integers
  SEED(1) through SEED(7). These get and put functions are slower
  than FM_RANDOM_NUMBER, so FM_RANDOM_NUMBER should be called many
  times between FM_RANDOM_SEED_PUT calls. Also, some generators that
  used a 9-digit modulus have failed randomness tests when used with
  only a few numbers being generated between calls to re-start with
  a new seed.

  FM_RANDOM_SEED_SIZE(SIZE)  returns integer SIZE as the size of the SEED array used by the
  FM_RANDOM_NUMBER generator. Currently, SIZE = 7.

  FM_RATIONAL_POWER(MA, K, J, MB)  \( MB = MA^{(K/J)} \) Rational power.
  Faster than \( MB = MA^{(\text{TO_FM}(K)/J)} \) for functions like the cube root.

  FM_READ(KREAD, MA)  MA is returned after reading one (possibly multi-line) FM number
  on unit KREAD. This routine reads numbers written by FM_WRITE.

  FM_SET(NPREC)  Set the internal FM variables so that the precision is at least
  NPREC base 10 digits plus three base 10 guard digits.

  FM_SETVAR(STRING)  Define a new value for one of the internal FM variables in module
FMVALS that controls one of the FM options. STRING has the form
variable = value.
Example: To change the screen width for FM output:
   CALL FM_SETVAR(' KSWIDE = 120 ')
The variables that can be changed and the options they control are
listed in sections 2 through 6 of the comments at the top of the
FM.f95 file. Only one variable can be set per call. The variable
name in STRING must have no embedded blanks. The value part of
STRING can be in any numerical format, except in the case of
variable CMCHAR, which is character type. To set CMCHAR to 'E',
don't use any quotes in STRING:
   CALL FM_SETVAR(' CMCHAR = E ')

FM_ULP(MA,MB) MB = One Unit in the Last Place of MA. For positive MA this is the
same as the Fortran function SPACING, but MB < 0 if MA < 0.
Examples: If MBASE = 10 and NDIG = 30, then ulp(1.0) =
1.0E-29, ulp(-4.5E+67) = -1.0E+38.

FM_VARS Write the current values of the internal FM variables on unit KW.

FM_WRITE(KWRITE,MA) Write MA on unit KWRITE. Multi-line numbers will have '&' as the last nonblank character
on all but the last line. These numbers can then be read easily
using FM_READ.

2. Type (IM). MA, MB, MC refer to type (IM) numbers.

IM_DIVR(MA,MB,MC,MD) MC = int(MA/MB), MD = MA mod MB
When both the quotient and remainder are needed, this routine
is twice as fast as doing MC = MA/MB and MD = MOD(MA,MB)
separately.

IM_DVIR(MA,IVAL,MB,IREM) MB = int(MA/IVAL), IREM = MA mod IVAL
IVAL and IREM are one word integers. Faster than doing separately.

IM_FORM(FORM,MA,STRING) MA is converted to a character string using format FORM and
returned in STRING. FORM can represent I, F, E, or ES formats.
Example: CALL IMFORM('I70',MA,STRING)

IM_FPRINT(FORM,MA) Print MA on unit KW using FORM format.

IM_PRINT(MA) Print MA on unit KW.

IM_READ(KREAD,MA) MA is returned after reading one (possibly multi-line) IM number
on unit KREAD. This routine reads numbers written by IM_WRITE.

IM_WRITE(KWRITE,MA) Write MA on unit KWRITE. Multi-line numbers will have '&' as the
last nonblank character on all but the last line. These numbers can then be read easily using IM_READ.

3. Type (ZM). MA, MB, MC refer to type (ZM) numbers. MBFM is type (FM).

ZM_ARG(MA,MBFM) MBFM = complex argument of MA. MBFM is the (real) angle in the
interval (-pi , pi ] from the positive real axis to the
point (x,y) when MA = x + y*i.
! ZM_COSH_SINH(MA,MB,MC)  MB = COSH(MA),  MC = SINH(MA).
! Faster than 2 calls.

! ZM_COS_SIN(MA,MB,MC)  MB = COS(MA),  MC = SIN(MA).
! Faster than 2 calls.

! ZM_FORM(FORM1,FORM2,MA,STRING)
! STRING = MA
! MA is converted to a character string using format FORM1 for the
! real part and FORM2 for the imaginary part. The result is returned
! in STRING. FORM1 and FORM2 can represent I, F, E, or ES formats.
! Example:
! CALL ZMFORM('F20.10','F15.10',MA,STRING)

! ZM_FPRINT(FORM1,FORM2,MA)  Print MA on unit KW using formats FORM1 and FORM2.

! ZM_PRINT(MA)  Print MA on unit KW using current format.

! ZM_READ(KREAD,MA)  MA is returned after reading one (possibly multi-line) ZM number
! on unit KREAD. This routine reads numbers written by ZMWRITE.

! ZM_RATIONAL_POWER(MA,IVAL,JVAL,MB)
! MB = MA ** (IVAL/JVAL)
! Faster than MB = MA**(TO_FM(K)/J) for functions like the cube root.

! ZM_WRITE(KWRITE,MA)  Write MA on unit KWRITE. Multi-line numbers are formatted for
! automatic reading with ZMREAD.

! Some other functions are defined that do not correspond to machine precision intrinsic
! functions. These include formatting functions, integer modular functions and GCD, and some
! mathematical special functions.
! N, K below are machine precision integers, J1, J2, J3 are TYPE (IM), FMT, FMTR, FMTI are
! character strings, A, B, X are TYPE (FM), and Z is TYPE (ZM).
! The three formatting functions return a character string containing the formatted number, the
! three TYPE (IM) functions return a TYPE (IM) result, and the 12 special functions return
! TYPE (FM) results.

! Formatting functions:

! FM_FORMAT(FMT,A)  Put A into FMT (real) format
! IM_FORMAT(FMT,J1)  Put J1 into FMT (integer) format
! ZM_FORMAT(FMTR,FMTI,Z)  Put Z into (complex) format, FMTR for the real
! part and FMTI for the imaginary part

! Examples:
! ST = FM_FORMAT('F65.60',A)
! WRITE (*,*) ' A = ',TRIM(ST)
! ST = FM_FORMAT('E75.60',B)
! WRITE (*,*) ' B = ',ST(1:75)
! ST = IM_FORMAT('IS0',J1)
! WRITE (*,*) ' J1 = ',ST(1:50)
! ST = ZM_FORMAT('F35.30','F30.25',Z)
! WRITE (*,*) ' Z = ',ST(1:70)

! These functions are used for one-line output. The returned character strings are of
! length 200.
For higher precision numbers, the output can be broken onto multiple lines automatically by calling subroutines FM_PRINT, IM_PRINT, ZM_PRINT, or the line breaks can be done by hand after calling one of the subroutines FM_FORM, IM_FORM, ZM_FORM.

For ZM_FORMAT the length of the output is 5 more than the sum of the two field widths.

Integer functions:

- **BINOMIAL(N,K)**: Binomial coefficient N choose K. Returns the exact result as a type IM value.
- **BINOMIAL(J1,J2)**: Binomial coefficient J1 choose J2. Like factorial below, the result might be too large unless min(J2,J1-J2) is fairly small.
- **FACTORIAL(N)**: N! Returns the exact result as a type IM value.
- **FACTORIAL(J1)**: J1! Note that the factorial function grows so rapidly that if type IM variable J1 is larger than the largest machine precision integer, then J1! has over 10 billion digits and the calculation would likely fail due to memory or time constraints. This version is provided for convenience, and will return UNKNOWN if J1 cannot be represented as a machine precision integer.
- **GCD(J1,J2)**: Greatest Common Divisor of J1 and J2.
- **MULTIPLY_MOD(J1,J2,J3)**: J1 * J2 mod J3
- **POWER_MOD(J1,J2,J3)**: J1 ** J2 mod J3

Special functions:

- **BERNOULLI(N)**: Nth Bernoulli number
- **BESSEL_J(N,X)**: Bessel function of the first kind J_n(x)
- **BESSEL_J0(X)**: Fortran-08 name for J_0(x)
- **BESSEL_J1(X)**: Fortran-08 name for J_1(x)
- **BESSEL_JN(N,X)**: Fortran-08 name for J_n(x)
- **BESSEL_JN(N1,N2,X)**: Returns array (/ J_n1(x) , ... , J_n2(x) /)
- **BESSEL_Y(N,X)**: Bessel function of the second kind Y_n(x)
- **BESSEL_Y0(X)**: Fortran-08 name for Y_0(x)
- **BESSEL_Y1(X)**: Fortran-08 name for Y_1(x)
- **BESSEL_YN(N,X)**: Fortran-08 name for Y_n(x)
- **BESSEL_YN(N1,N2,X)**: Returns array (/ Y_n1(x) , ... , Y_n2(x) /)
- **BETA(A,B)**: Integral (0 to 1) t**(a-1) * (1-t)**(b-1) dt
- **BINOMIAL(A,B)**: Binomial Coefficient a! / ( b! (a-b)! )
- **COS_INTEGRAL(X)**: Cosine Integral Ci(x)
- **COSH_INTEGRAL(X)**: Hyperbolic Cosine Integral Chi(x)
- **ERF(X)**: Error function Erf(x)
- **ERFC(X)**: Complimentary error function Erfc(x)
- **ERFC_SCALED(X)**: Exp(x^2) * Erfc(x)
- **EXP_INTEGRAL_EI(X)**: Exponential Integral Ei(x)
- **EXP_INTEGRAL_EN(N,X)**: Exponential Integral E_n(x)
- **FACTORIAL(X)**: x! = Gamma(x+1)
- **FRESNEL_C(X)**: Fresnel Cosine Integral C(x)
- **FRESNEL_S(X)**: Fresnel Sine Integral S(x)
- **FRESNEL_S(X)**: Fresnel Sine Integral S(x)
- **GAMMA(X)**: Integral (0 to infinity) t**(x-1) * exp(-t) dt
- **INCOMPLETE_BETA(X,A,B)**: Integral (0 to x) t**(a-1) * (1-t)**(b-1) dt
- **INCOMPLETE_GAMMA1(A,X)**: Integral (0 to x) t**(a-1) * exp(-t) dt
- **INCOMPLETE_GAMMA2(A,X)**: Integral (x to infinity) t**(a-1) * exp(-t) dt
- **LOG_ERFC(X)**: Ln( Erfc(X) ).
- **LOG_GAMMA(X)**: Analytic continuation of real Ln( Gamma(x) ). May differ from complex Ln( Gamma(x) ) by an integer multiple of 2*pi*i.
- **LOG_INTEGRAL(X)**: Logarithmic Integral Li(x)
- **POCHHAMMER(X,N)**: x*(x+1)*(x+2)*...*(x+n-1)
POLYGAMMA(N,X)  Nth derivative of Psi(x)
PSI(X)         Derivative of Ln(Gamma(x))
SIN_INTEGRAL(X) Sine Integral Si(x)
SINH_INTEGRAL(X) Hyperbolic Sine Integral Shi(x)

Array operations:

Arithmetic operations and functions on arrays of dimension (rank) one or two are supported for each of the three multiple-precision types. Binary operations (+-*/) require both arguments to have the same rank and shape.

Examples:

```fortran
!TYPE (FM), SAVE, DIMENSION(10) :: A, B
!TYPE (FM), SAVE, DIMENSION(3,3) :: C
!TYPE (IM), SAVE, DIMENSION(10) :: J, K
!TYPE (IM), SAVE, DIMENSION(3,3) :: L
!
!A = 0                               ! Set the whole array to zero
!J = J * K                           ! Set J(i) = J(i) * K(i) for i = 1, ..., 10
!B = A - K                           ! Mixed-mode operations are ok
!C = 7.3D0 * C - (C + 2*L )/3
```

Array functions:

```fortran
DOT_PRODUCT(X,Y) Dot product of rank 1 vectors of the same type.
Note that when X and Y are complex, the result is not just the sum of the products of the corresponding array elements, as it is for types FM and IM. For ZM the formula is the sum of conjg(X(j)) * Y(j).
IS_OVERFLOW(X) Returns true if any element is + or - overflow.
IS_UNDERFLOW(X) Returns true if any element is + or - underflow.
IS_UNKNOWN(X) Returns true if any element is unknown.
MATMUL(X,Y) Matrix multiplication of arrays of the same type.
Cases for valid argument shapes:
(1) (n,m) * (m,k) --> (n,k)
(2) (m) * (m,k) --> (k)
(3) (n,m) * (m) --> (n)
MAXLOC(X) Location of the maximum value in the array
MAXVAL(X) Maximum value in the array
MINLOC(X) Location of the minimum value in the array
MINVAL(X) Minimum value in the array
PRODUCT(X) Product of all values in the array
SUM(X) Sum of all values in the array
TRANSPOSE(X) Matrix transposition. If X is a rank 2 array with shape (n,m), then Y = TRANSPOSE(X) has shape (m,n) with Y(i,j) = X(j,i).
TO_FM(X) Rank 1 or 2 arrays are converted to similar type (fm) arrays.
TO_IM(X) Rank 1 or 2 arrays are converted to similar type (im) arrays.
TO_ZM(X) Rank 1 or 2 arrays are converted to similar type (zm) arrays.
TO_INT(X) Rank 1 or 2 arrays are converted to similar integer arrays.
TO_SP(X) Rank 1 or 2 arrays are converted to similar single precision arrays.
TO_DP(X) Rank 1 or 2 arrays are converted to similar double precision arrays.
TO_SPZ(X) Rank 1 or 2 arrays are converted to similar single complex arrays.
TO_DPZ(X) Rank 1 or 2 arrays are converted to similar double complex arrays.
```

The arithmetic array functions DOT_PRODUCT, MATMUL, PRODUCT, and SUM work like the other functions in the FM package in that they raise precision and compute the sums and/or products at the higher precision, then round the final result back to the user's precision to provide...
a more accurate result.

Fortran's optional [,mask] argument for these functions is not provided.

Many of the 1-argument functions can be used with array arguments, with the result being an array of the same size and shape where the function has been applied to each element.

Examples:

```fortran
TYPE (FM), SAVE, DIMENSION(10) :: A, B, C
... 
A = ABS(B)  ! Set A(i) = ABS(B(i)) for i = 1, ..., 10
C = SQRT(A+4+B*B)  ! Set C(i) = SQRT(A(i)+4+B(i)*B(i)) for i = 1, ..., 10
```

Functions that can have array arguments. As above, "real", "integer", and "complex" refer to types FM, IM, and ZM respectively.

```
ABS              real   integer   complex
ACOS             real               complex
ACOSH            real               complex
AIMAG                               complex
AINT             real              complex
ANINT            real               complex
ASIN             real               complex
ASINH            real               complex
ATAN             real               complex
ATANH            real               complex
CEILING          real    integer    complex
CONJG                               complex
COS              real               complex
COSH             real               complex
EXP              real              complex
FLOOR            real   integer    complex
FRACTION         real               complex
INT              real    integer    complex
LOG              real               complex
LOG10            real               complex
NINT             real    integer    complex
SIN              real               complex
SINH             real               complex
SQRT             real               complex
TAN              real               complex
TANH             real               complex
COS_INTEGRAL     real               complex
COSH_INTEGRAL    real               complex
ERF              real               complex
ERFC             real               complex
ERFC_SCALED      real               complex
EXP_INTEGRAL_EI  real               complex
FACTORIAL        real    integer    complex    machine-precision integer
FRESNEL_C        real               complex
FRESNEL_S        real               complex
GAMMA            real               complex
LOG_ERFC         real               complex
LOG_GAMMA        real               complex
LOG_INTEGRAL     real               complex
PSI              real               complex
SIN_INTEGRAL     real               complex
SINH_INTEGRAL    real               complex
```
INTERFACE TO_FM
  MODULE PROCEDURE FM_I
  MODULE PROCEDURE FM_R
  MODULE PROCEDURE FM_D
  MODULE PROCEDURE FM_Z
  MODULE PROCEDURE FM_ZD
  MODULE PROCEDURE FM_IM
  MODULE PROCEDURE FM_FM
  MODULE PROCEDURE FM_ST
  MODULE PROCEDURE FM_I1
  MODULE PROCEDURE FM_R1
  MODULE PROCEDURE FM_D1
  MODULE PROCEDURE FM_Z1
  MODULE PROCEDURE FM_ZD1
  MODULE PROCEDURE FM_FM1
  MODULE PROCEDURE FM_IM1
  MODULE PROCEDURE FM_ZM1
  MODULE PROCEDURE FM_ST1
  MODULE PROCEDURE FM_I2
  MODULE PROCEDURE FM_R2
  MODULE PROCEDURE FM_D2
  MODULE PROCEDURE FM_Z2
  MODULE PROCEDURE FM_ZD2
  MODULE PROCEDURE FM_FM2
  MODULE PROCEDURE FM_IM2
  MODULE PROCEDURE FM_ZM2
  MODULE PROCEDURE FM_ST2
END INTERFACE

INTERFACE TO_IM
  MODULE PROCEDURE IM_I
  MODULE PROCEDURE IM_R
  MODULE PROCEDURE IM_D
  MODULE PROCEDURE IM_Z
  MODULE PROCEDURE IM_C
  MODULE PROCEDURE IM_FM
  MODULE PROCEDURE IM_IM
  MODULE PROCEDURE IM_ZM
  MODULE PROCEDURE IM_ST
  MODULE PROCEDURE IM_I1
  MODULE PROCEDURE IM_R1
  MODULE PROCEDURE IM_D1
  MODULE PROCEDURE IM_Z1
  MODULE PROCEDURE IM_C1
  MODULE PROCEDURE IM_FM1
  MODULE PROCEDURE IM_IM1
  MODULE PROCEDURE IM_ZM1
  MODULE PROCEDURE IM_ST1
  MODULE PROCEDURE IM_I2
  MODULE PROCEDURE IM_R2
  MODULE PROCEDURE IM_D2
  MODULE PROCEDURE IM_Z2
  MODULE PROCEDURE IM_C2
  MODULE PROCEDURE IM_FM2
  MODULE PROCEDURE IM_IM2
MODULE PROCEDURE IM_ZM2
MODULE PROCEDURE IM_ST2
END INTERFACE

INTERFACE TO_ZM
  MODULE PROCEDURE ZM_I
  MODULE PROCEDURE ZM2_I
  MODULE PROCEDURE ZM_R
  MODULE PROCEDURE ZM2_R
  MODULE PROCEDURE ZM_D
  MODULE PROCEDURE ZM2_D
  MODULE PROCEDURE ZM_Z
  MODULE PROCEDURE ZM_C
  MODULE PROCEDURE ZM_FM
  MODULE PROCEDURE ZM_IM
  MODULE PROCEDURE ZM_ZM
  MODULE PROCEDURE ZM_ST
  MODULE PROCEDURE ZM_I1
  MODULE PROCEDURE ZM_R1
  MODULE PROCEDURE ZM_D1
  MODULE PROCEDURE ZM_Z1
  MODULE PROCEDURE ZM_C1
  MODULE PROCEDURE ZM_FM1
  MODULE PROCEDURE ZM_IM1
  MODULE PROCEDURE ZM_ZM1
  MODULE PROCEDURE ZM_ST1
  MODULE PROCEDURE ZM_I2
  MODULE PROCEDURE ZM_R2
  MODULE PROCEDURE ZM_D2
  MODULE PROCEDURE ZM_Z2
  MODULE PROCEDURE ZM_C2
  MODULE PROCEDURE ZM_FM2
  MODULE PROCEDURE ZM_IM2
  MODULE PROCEDURE ZM_ZM2
  MODULE PROCEDURE ZM_ST2
END INTERFACE

INTERFACE TO_INT
  MODULE PROCEDURE FM_2INT
  MODULE PROCEDURE IM_2INT
  MODULE PROCEDURE ZM_2INT
  MODULE PROCEDURE FM_2INT1
  MODULE PROCEDURE IM_2INT1
  MODULE PROCEDURE ZM_2INT1
  MODULE PROCEDURE FM_2INT2
  MODULE PROCEDURE IM_2INT2
  MODULE PROCEDURE ZM_2INT2
END INTERFACE

INTERFACE TO_SP
  MODULE PROCEDURE FM_2SP
  MODULE PROCEDURE IM_2SP
  MODULE PROCEDURE ZM_2SP
  MODULE PROCEDURE FM_2SP1
  MODULE PROCEDURE IM_2SP1
  MODULE PROCEDURE ZM_2SP1
  MODULE PROCEDURE FM_2SP2
  MODULE PROCEDURE IM_2SP2
  MODULE PROCEDURE ZM_2SP2
END INTERFACE
MODULE  PROCEDURE  ZM_ZSP2  
END INTERFACE

INTERFACE  TO_DP  
MODULE  PROCEDURE  FM_ZDP  
MODULE  PROCEDURE  IM_ZDP  
MODULE  PROCEDURE  ZM_ZDP  
MODULE  PROCEDURE  FM_ZDP1  
MODULE  PROCEDURE  IM_ZDP1  
MODULE  PROCEDURE  ZM_ZDP1  
MODULE  PROCEDURE  FM_ZDP2  
MODULE  PROCEDURE  IM_ZDP2  
MODULE  PROCEDURE  ZM_ZDP2  
END INTERFACE

INTERFACE  TO_SP  
MODULE  PROCEDURE  FM_ZSP  
MODULE  PROCEDURE  IM_ZSP  
MODULE  PROCEDURE  ZM_ZSP  
MODULE  PROCEDURE  FM_ZSP1  
MODULE  PROCEDURE  IM_ZSP1  
MODULE  PROCEDURE  ZM_ZSP1  
MODULE  PROCEDURE  FM_ZSP2  
MODULE  PROCEDURE  IM_ZSP2  
MODULE  PROCEDURE  ZM_ZSP2  
END INTERFACE

INTERFACE  TO_DP  
MODULE  PROCEDURE  FM_ZDP  
MODULE  PROCEDURE  IM_ZDP  
MODULE  PROCEDURE  ZM_ZDP  
MODULE  PROCEDURE  FM_ZDP1  
MODULE  PROCEDURE  IM_ZDP1  
MODULE  PROCEDURE  ZM_ZDP1  
MODULE  PROCEDURE  FM_ZDP2  
MODULE  PROCEDURE  IM_ZDP2  
MODULE  PROCEDURE  ZM_ZDP2  
END INTERFACE

INTERFACE  IS_OVERFLOW  
MODULE  PROCEDURE  FM_IS_OVERFLOW  
MODULE  PROCEDURE  IM_IS_OVERFLOW  
MODULE  PROCEDURE  ZM_IS_OVERFLOW  
MODULE  PROCEDURE  FM_IS_OVERFLOW1  
MODULE  PROCEDURE  IM_IS_OVERFLOW1  
MODULE  PROCEDURE  ZM_IS_OVERFLOW1  
MODULE  PROCEDURE  FM_IS_OVERFLOW2  
MODULE  PROCEDURE  IM_IS_OVERFLOW2  
MODULE  PROCEDURE  ZM_IS_OVERFLOW2  
END INTERFACE

INTERFACE  IS_UNDERFLOW  
MODULE  PROCEDURE  FM_IS_UNDERFLOW  
MODULE  PROCEDURE  IM_IS_UNDERFLOW  
MODULE  PROCEDURE  ZM_IS_UNDERFLOW  
MODULE  PROCEDURE  FM_IS_UNDERFLOW1  
MODULE  PROCEDURE  IM_IS_UNDERFLOW1  
MODULE  PROCEDURE  ZM_IS_UNDERFLOW1  
END INTERFACE
MODULE PROCEDURE FM_IS_UNDERFLOW2
MODULE PROCEDURE IM_IS_UNDERFLOW2
MODULE PROCEDURE ZM_IS_UNDERFLOW2
END INTERFACE

INTERFACE IS_UNKNOWN
MODULE PROCEDURE FM_IS_UNKNOWN
MODULE PROCEDURE IM_IS_UNKNOWN
MODULE PROCEDURE ZM_IS_UNKNOWN
MODULE PROCEDURE FM_IS_UNKNOWN1
MODULE PROCEDURE IM_IS_UNKNOWN1
MODULE PROCEDURE ZM_IS_UNKNOWN1
MODULE PROCEDURE FM_IS_UNKNOWN2
MODULE PROCEDURE IM_IS_UNKNOWN2
MODULE PROCEDURE ZM_IS_UNKNOWN2
END INTERFACE

INTERFACE FM_UNDEF_INP
MODULE PROCEDURE FM_UNDEF_INP_FM0
MODULE PROCEDURE FM_UNDEF_INP_IM0
MODULE PROCEDURE FM_UNDEF_INP_ZM0
MODULE PROCEDURE FM_UNDEF_INP_FM1
MODULE PROCEDURE FM_UNDEF_INP_IM1
MODULE PROCEDURE FM_UNDEF_INP_ZM1
MODULE PROCEDURE FM_UNDEF_INP_FM2
MODULE PROCEDURE FM_UNDEF_INP_IM2
MODULE PROCEDURE FM_UNDEF_INP_ZM2
END INTERFACE

! The next function is no longer needed in version 1.4.
! Dummy versions of the individual procedures are included for compatibility with version 1.3.

INTERFACE FM_DEALLOCATE
MODULE PROCEDURE FM_DEALLOCATE_FM1
MODULE PROCEDURE FM_DEALLOCATE_IM1
MODULE PROCEDURE FM_DEALLOCATE_ZM1
MODULE PROCEDURE FM_DEALLOCATE_FM2
MODULE PROCEDURE FM_DEALLOCATE_IM2
MODULE PROCEDURE FM_DEALLOCATE_ZM2
END INTERFACE

CONTAINS

! TO_FM

FUNCTION FM_I(IVAL) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPlicit NONE
  TYPE (FM) :: RETURN_VALUE
  INTEGER :: IVAL
  INTENT (IN) :: IVAL
  CALL FMI2M(IVAL,RETURN_VALUE%MFM)
END FUNCTION FM_I

FUNCTION FM_R(R) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPlicit NONE
FUNCTION FM_R
  REAL :: R
  INTENT (IN) :: R
  CALL FMSP2M(R,RETURN_VALUE%MFM)
END FUNCTION FM_R

FUNCTION FM_D(D) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  DOUBLE PRECISION :: D
  INTENT (IN) :: D
  CALL FMDP2M(D,RETURN_VALUE%MFM)
END FUNCTION FM_D

FUNCTION FM_Z(Z) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  COMPLEX :: Z
  INTENT (IN) :: Z
  CALL FMSP2M(REAL(Z),RETURN_VALUE%MFM)
END FUNCTION FM_Z

FUNCTION FM_ZD(C) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  COMPLEX :: C
  INTENT (IN) :: C
  CALL FMDP2M(REAL(C,KIND(0.0D0)),RETURN_VALUE%MFM)
END FUNCTION FM_ZD

FUNCTION FM_FM(MA) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE,MA
  INTENT (IN) :: MA
  CALL FM_UNDEF_INP(MA)
  CALL FMEQ(MA%MFM,RETURN_VALUE%MFM)
END FUNCTION FM_FM

FUNCTION FM_IM(MA) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  TYPE (IM) :: MA
  INTENT (IN) :: MA
  CALL FM_UNDEF_INP(MA)
  CALL IMI2FM(MA%MIM,RETURN_VALUE%MFM)
END FUNCTION FM_IM

FUNCTION FM_ZM(MA) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  TYPE (ZM) :: MA
  INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
CALL ZMREAL(MA%MZM,RETURN_VALUE%MFM)
END FUNCTION FM_ZM

FUNCTION FM_ST(ST) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  TYPE (FM) :: RETURN_VALUE
  CHARACTER(*) :: ST
  INTENT (IN) :: ST
  CALL FMST2M(ST,RETURN_VALUE%MFM)
END FUNCTION FM_ST

FUNCTION FM_I1(IVAL) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  INTEGER, DIMENSION(:) :: IVAL
  TYPE (FM), DIMENSION(SIZE(IVAL)) :: RETURN_VALUE
  INTEGER :: J,N
  INTENT (IN) :: IVAL
  N = SIZE(IVAL)
  DO J = 1, N
    CALL FMINT2M(IVAL(J),RETURN_VALUE(J)%MFM)
  ENDDO
END FUNCTION FM_I1

FUNCTION FM_R1(R) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  REAL, DIMENSION(:) :: R
  TYPE (FM), DIMENSION(SIZE(R)) :: RETURN_VALUE
  INTEGER :: J,N
  INTENT (IN) :: R
  N = SIZE(R)
  DO J = 1, N
    CALL FMSP2M(R(J),RETURN_VALUE(J)%MFM)
  ENDDO
END FUNCTION FM_R1

FUNCTION FM_D1(D) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  DOUBLE PRECISION, DIMENSION(:) :: D
  TYPE (FM), DIMENSION(SIZE(D)) :: RETURN_VALUE
  INTEGER :: J,N
  INTENT (IN) :: D
  N = SIZE(D)
  DO J = 1, N
    CALL FMDP2M(D(J),RETURN_VALUE(J)%MFM)
  ENDDO
END FUNCTION FM_D1

FUNCTION FM_Z1(Z) RESULT (RETURN_VALUE)
  USE FMVALS
  IMPLICIT NONE
  COMPLEX, DIMENSION(:) :: Z
  TYPE (FM), DIMENSION(SIZE(Z)) :: RETURN_VALUE
  INTEGER :: J,N
INTENT (IN) :: Z
N = SIZE(Z)
DO J = 1, N
    CALL FMSP2M(REAL(Z(J)),RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_Z1

FUNCTION FM_ZD1(C) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
COMPLEX (KIND(0.0D0)), DIMENSION(:) :: C
TYPE (FM), DIMENSION(SIZE(C)) :: RETURN_VALUE
INTEGER :: J,N
INTENT (IN) :: C
N = SIZE(C)
DO J = 1, N
    CALL FMDP2M(REAL(C(J),KIND(0.0D0)),RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_ZD1

FUNCTION FM_FM1(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (FM), DIMENSION(:) :: MA
TYPE (FM), DIMENSION(SIZE(MA)) :: RETURN_VALUE
INTEGER :: J,N
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
N = SIZE(MA)
DO J = 1, N
    CALL FMEQ(MA(J)%MFM,RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_FM1

FUNCTION FM_IM1(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (IM), DIMENSION(:) :: MA
TYPE (FM), DIMENSION(SIZE(MA)) :: RETURN_VALUE
INTEGER :: J,N
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
N = SIZE(MA)
DO J = 1, N
    CALL IMI2FM(MA(J)%MIM,RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_IM1

FUNCTION FM_ZM1(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (ZM), DIMENSION(:) :: MA
TYPE (FM), DIMENSION(SIZE(MA)) :: RETURN_VALUE
INTEGER :: J,N
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
N = SIZE(MA)
DO J = 1, N
    CALL ZMIF2FM(MA(J)%MFM,RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_ZM1
CALL ZMREAL(MA(J)%MZM,RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_ZM1

FUNCTION FM_ST1(ST) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
CHARACTER(*) :: ST
TYPE (FM) :: RETURN_VALUE
INTEGER :: J,N
INTENT (IN) :: ST
N = SIZE(ST)
DO J = 1, N
  CALL FMST2M(ST(J),RETURN_VALUE(J)%MFM)
ENDDO
END FUNCTION FM_ST1

FUNCTION FM_I2(IVAL) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
INTEGER :: J,K
INTENT (IN) :: IVAL
DO J = 1, SIZE(IVAL,DIM=1)
  DO K = 1, SIZE(IVAL,DIM=2)
    CALL FMI2M(IVAL(J,K),RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_I2

FUNCTION FM_R2(R) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
REAL :: R
INTENT (IN) :: R
DO J = 1, SIZE(R,DIM=1)
  DO K = 1, SIZE(R,DIM=2)
    CALL FMSP2M(R(J,K),RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_R2

FUNCTION FM_D2(D) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
DOUBLE PRECISION :: D
INTENT (IN) :: D
DO J = 1, SIZE(D,DIM=1)
  DO K = 1, SIZE(D,DIM=2)
    CALL FMDP2M(D(J,K),RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_D2
FUNCTION FM_Z2(Z) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
COMPLEX, DIMENSION(:, :) :: Z
TYPE (FM), DIMENSION(SIZE(Z,DIM=1),SIZE(Z,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: Z
DO J = 1, SIZE(Z,DIM=1)
  DO K = 1, SIZE(Z,DIM=2)
    CALL FMSP2M(REAL(Z(J,K)),RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_Z2

FUNCTION FM_ZD2(C) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
COMPLEX (KIND(0.0D0)), DIMENSION(:, :) :: C
TYPE (FM), DIMENSION(SIZE(C,DIM=1),SIZE(C,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: C
DO J = 1, SIZE(C,DIM=1)
  DO K = 1, SIZE(C,DIM=2)
    CALL FMDP2M(REAL(C(J,K),KIND(0.0D0)),RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_ZD2

FUNCTION FM_FM2(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (FM), DIMENSION(:, :) :: MA
TYPE (FM), DIMENSION(SIZE(MA,DIM=1),SIZE(MA,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
DO J = 1, SIZE(MA,DIM=1)
  DO K = 1, SIZE(MA,DIM=2)
    CALL FMEQ(MA(J,K)%MFM,RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_FM2

FUNCTION FM_IM2(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (FM), DIMENSION(:, :) :: MA
TYPE (FM), DIMENSION(SIZE(MA,DIM=1),SIZE(MA,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
DO J = 1, SIZE(MA,DIM=1)
  DO K = 1, SIZE(MA,DIM=2)
    CALL IMI2FM(MA(J,K)%MIM,RETURN_VALUE(J,K)%MFM)
  ENDDO
ENDDO
END FUNCTION FM_IM2
FUNCTION FM_ZM2(MA) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (ZM), DIMENSION(::,:) :: MA
TYPE (FM), DIMENSION(SIZE(MA,DIM=1),SIZE(MA,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: MA
CALL FM_UNDEF_INP(MA)
DO J = 1, SIZE(MA,DIM=1)
   DO K = 1, SIZE(MA,DIM=2)
      CALL ZMREAL(MA(J,K)%MZM,RETURN_VALUE(J,K)%MFM)
   ENDDO
ENDDO
END FUNCTION FM_ZM2

FUNCTION FM_ST2(ST) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
CHARACTER(*), DIMENSION(::,:) :: ST
TYPE (FM), DIMENSION(SIZE(ST,DIM=1),SIZE(ST,DIM=2)) :: RETURN_VALUE
INTEGER :: J,K
INTENT (IN) :: ST
DO J = 1, SIZE(ST,DIM=1)
   DO K = 1, SIZE(ST,DIM=2)
      CALL FMST2M(ST(J,K),RETURN_VALUE(J,K)%MFM)
   ENDDO
ENDDO
END FUNCTION FM_ST2

FUNCTION IM_I(IVAL) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (IM) :: RETURN_VALUE
INTEGER :: IVAL
INTENT (IN) :: IVAL
CALL IMI2M(IVAL,RETURN_VALUE%MIM)
END FUNCTION IM_I

FUNCTION IM_R(R) RESULT (RETURN_VALUE)
USE FMVALS
IMPLICIT NONE
TYPE (IM) :: RETURN_VALUE
REAL :: R
CHARACTER(25) :: ST
INTEGER :: IVAL
INTENT (IN) :: R
IF (ABS(R) < HUGE(1)) THEN
   IVAL = INT(R)
   CALL IMI2M(IVAL,RETURN_VALUE%MIM)
ELSE
   WRITE (ST,'(E25.16)') R
   CALL IMST2M(ST,RETURN_VALUE%MIM)
ENDIF
END FUNCTION IM_R