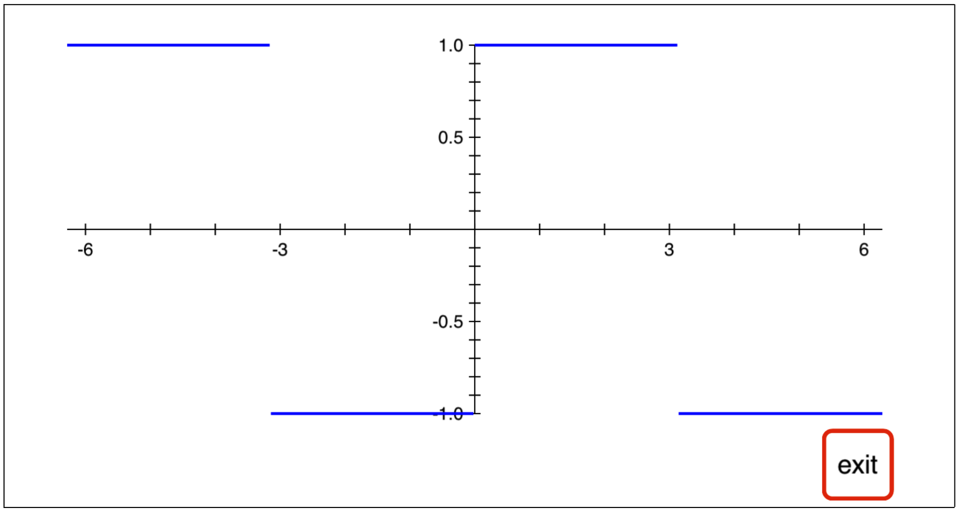
The Fourier series for a square wave is

$$f(t) = \frac{4}{\pi}\left(\frac{\sin(t)}{1} + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \cdots\right)$$

The graph is constant on intervals of length π, jumping between -1 and 1.



Define a function that will compute the partial sum of this series out to the $\sin(n\,t)/n$ term, and then graph some of these approximations to the square wave. By stopping the sum after a finite number of terms, we will get a continuous function that is trying to approximate a discontinuous one.

Function f1 will have input k and output $\sin(k\,t)/k$, so it will do one term of the Fourier sum.

Function f2 will store its input t in register 2, so it will be available for f1. To specify the number of terms of the series to use, let n be the final value of k to use for a partial sum of the series. Because the plot key needs a function of only one variable, we will store n in register 0 before calling f2.

f1: 1, sto, 2, rcl, *, sin, 1, rcl, /

f2: 2, sto, 1, enter, 0, rcl, 2, enter, 1, sum, 4, *, π, /

So f2 puts t into register 2 and uses the sum key to add
f1(1) + f1(3) + f1(5) + $\cdots$ + f1(n), then multiplies by $4/\pi$.

Now to add the terms out to $\sin(5\,t)/5$, first put 5 in register 0,

5, enter, 0, sto

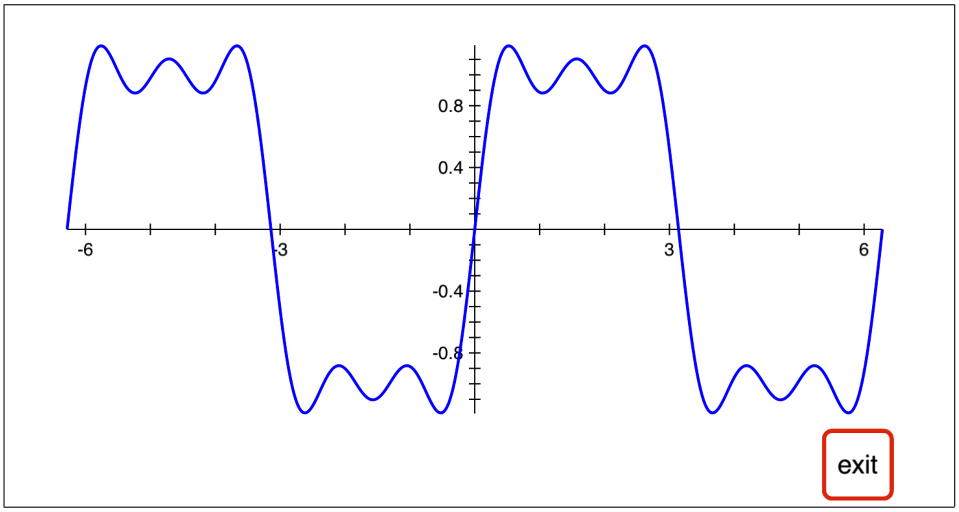
then sum the first three terms of the series at t = 0.3

.3, enter, 2, $\mathrm{f_n}$

which gives 0.9627... as the 3-term Fourier approximation to the square wave at t = 0.3.

Next, use the plot key to look at some of these approximations. We have $n = 5$ already stored in register 0, so this will show $f(t) = (4/\pi)\,(\sin(t)/1 + \sin(3t)/3 + \sin(5t)/5)$.

π, -2, *, π, 2, *, 2, plot

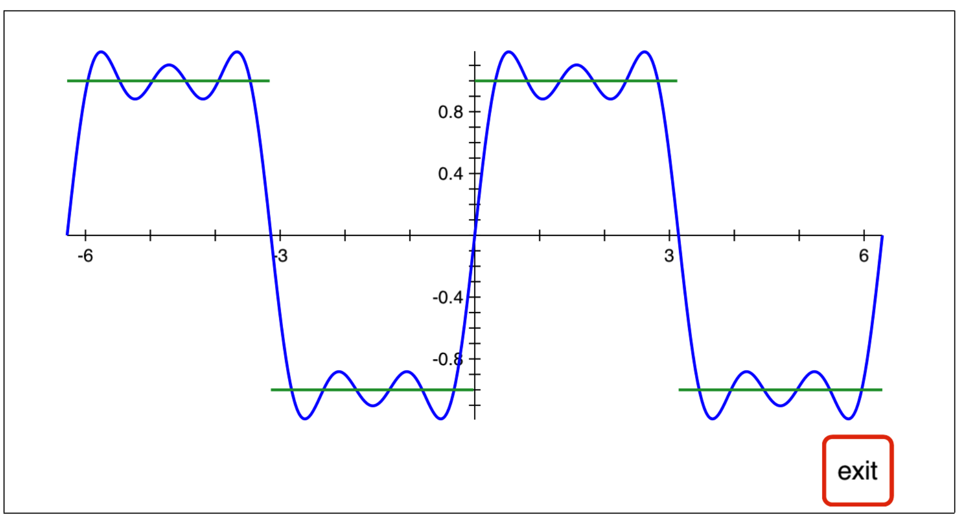


On the interval $(0,2\pi)$ the square wave function is 1 from 0 to π, then -1 from π to 2π, and extended periodically in both directions like $\sin(x)$. There are lots of possible ways to define it — one uses the floor function to give 0 when $\sin(x) \geq 0$ and -1 when $\sin(x) < 0$. Then apply $2x + 1$ to transform 0 to 1 and -1 to -1, defining f3 as the square wave function. Multiplying the $\sin(x)$ value by 0.9 avoids the possibility that $\sin(x)$ might round to 1.0 at one of the plotted points, giving the wrong value after doing floor, 2, *, 1, +.

f3: sin, .9, *, floor, 2, *, 1, +

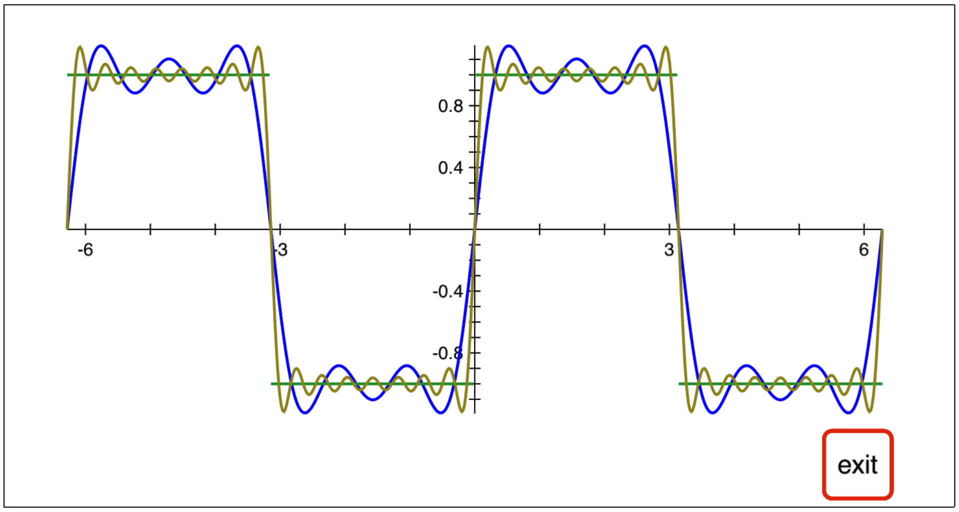
Add the graph of the square wave to the $n = 5$ plot.

π, -2, *, π, 2, *, 3, add plot



To look at the convergence of the Fourier series, add another plot with $n = 15$.

15, enter, 0, sto, π, -2, *, π, 2, *, 2, add plot



Because we added a plot of function 2 to a list of plots that already had another plot of function 2, trying to zoom in or zoom out from this plot will lose the first function 2 graph. If we wanted the zoom keys to keep all the graphs, we could have defined two different functions for the $n = 5$ and $n = 15$ cases and plotted them with different function numbers.

The reason for plotting the $n = 5$ case before adding the square wave plot is that when adding plots, the first plot sets the viewing window, so plotting the square wave first would cut off the $n = 5$ and $n = 15$ graphs whenever they were above $y = 1$ or below $y = -1$.