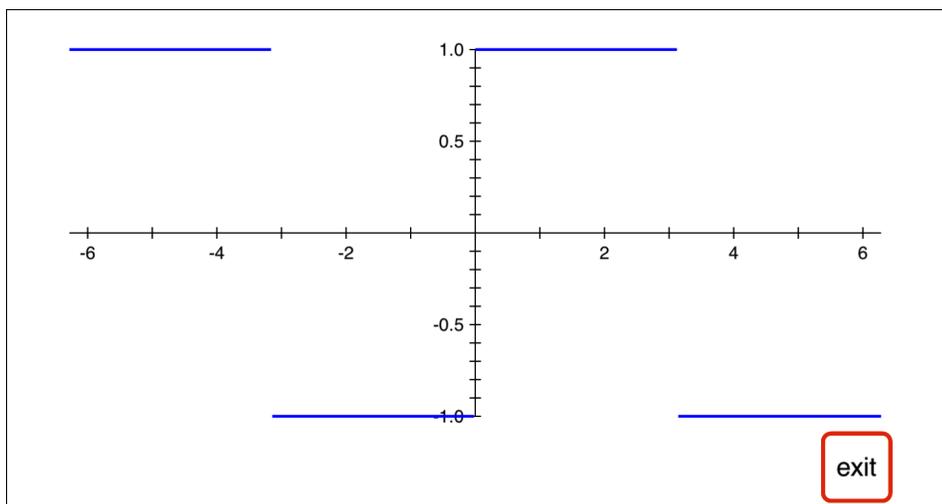


Graphics with Fourier Series

The Fourier series for a square wave is

$$f(t) = \frac{4}{\pi} \left(\frac{\sin(t)}{1} + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \dots \right)$$

The graph is constant on intervals of length π , jumping between -1 and 1 .



Define a function that will compute the partial sum of this series out to the $\sin(nt)/n$ term, and then graph some of these approximations to the square wave. By stopping the sum after a finite number of terms, we will get a continuous function that is trying to approximate a discontinuous one.

Function f1 will have input k and output $\sin(kt)/k$, so it will do one term of the Fourier sum.

Function f2 will store its input t in register 2, so it will be available for f1. To specify the number of terms of the series to use, let n be the final value of k to use for a partial sum of the series. Because the plot key needs a function of only one variable, we will store n in register 0 before calling f2.

For those new to Calc-50 programming, the “prog” key on screen 6 is used to begin defining a function. There are 10 possible functions, called f0, f1, ..., f9. Pressing this sequence of keys will let us begin the f1 definition:

6, func, 1, prog

If there is a previously defined function f1, then press the “clr” key to clear the old function and start a new one. The “sin” function in f1 is on screen 1, and when we start the function definition we are on screen 6, so we press “1, func” during the function definition to get to screen 1. That does not get saved as part of the function definition.

When we are done entering the function definition, press “6, func, end” to return to screen 6 and end the function definition. That will return to normal calculator mode.

For entering two numbers in succession the “enter” key ends the first number so we can start the next one. If an operation, like “rcl” or “sin”, follows a number, we don’t need to press “enter”.

When showing the keys to be pressed in order to to define functions like f1 and f2 below, we will omit the “6, func, 1, prog, clrf” or “6, func, 2, prog, clrf” at the start and the “6, func, end” at the end.

The multiplication key on the calculator, “ \times ”, is shown as “*” in the function definitions in these example pages, and also in the function definitions displayed on the calculator screen while programming functions.

f1: 1, func, 1, sto, 2, rcl, *, sin, 1, rcl, /

f2: 7, func, 2, sto, 1, enter, 0, rcl, 2, enter, 1, sum, 4, *, π , /

In the function definition itself there are commas between the numbers and commands, so the enter commands between numbers are omitted and the calculator displays function 2 as

f2: 2,sto,1,0,rcl,2,1,sum,4,*,pi,/_

The underscore at the end is the cursor showing where the next command will go. The four arrow keys on screen 6 move the cursor so a function can be edited.

The f2 function puts t into register 2 and uses the sum key with inputs 1, n , 2, 1 (0,rcl gets n). This means do a sum from 1 to n by steps of 2 of function f1: $f1(1) + f1(3) + f1(5) + \dots + f1(n)$. Then f2 multiplies that sum by $4/\pi$.

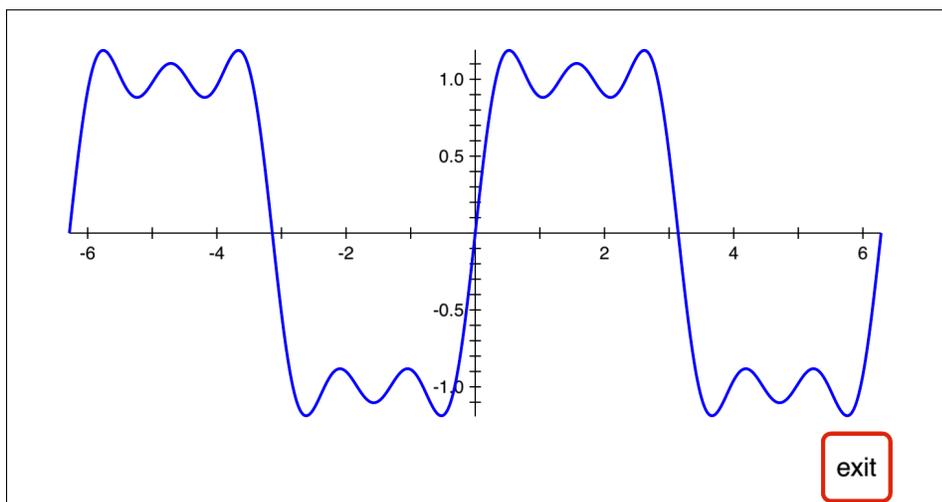
Add the terms out to $\sin(5t)/5$. Put $n = 5$ in register 0, then sum the first three terms at $t = 0.3$.

5, enter, 0, sto, 0.3, enter, 2, f_n

which gives 0.9627... as the 3-term Fourier approximation to the square wave at $t = 0.3$.

Next, use the plot key to look at some of these approximations. We have $n = 5$ already stored in register 0, so this will show $f(t) = (4/\pi) (\sin(t)/1 + \sin(3t)/3 + \sin(5t)/5)$. To enter the -2 below, press “2, chs”.

7, func, π , -2, *, π , 2, *, 2, plot



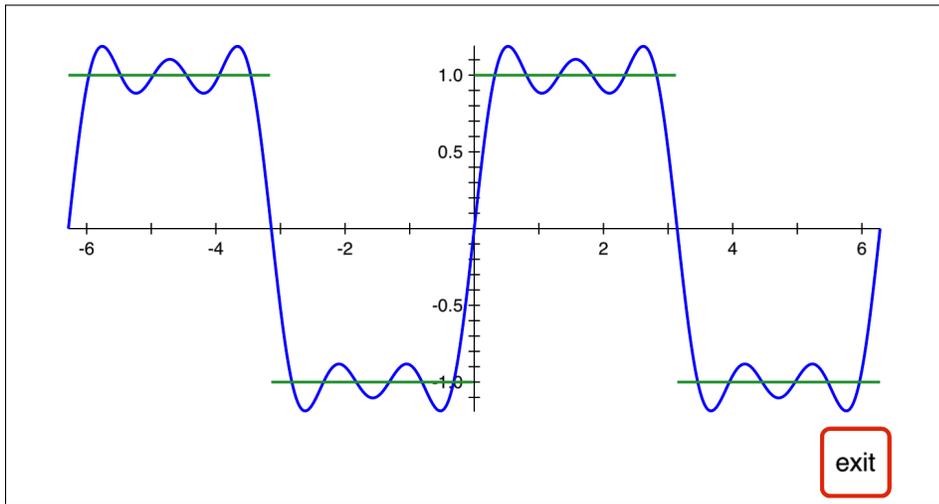
On the interval $(0, 2\pi)$ the square wave function is 1 from 0 to π , then -1 from π to 2π , and extended periodically in both directions like $\sin(x)$. There are lots of possible ways to define it — one uses the floor function to give 0 when $\sin(x) \geq 0$ and -1 when $\sin(x) < 0$. Then apply $2x + 1$ to transform 0 to 1 and -1 to -1 , defining f_3 as the square wave function. Multiplying $\sin(x)$ by 0.9 avoids the possibility that $\sin(x)$ might round to 1.0 at one of the plotted points, giving the wrong value after doing floor, 2, *, 1, +.

$$f_3(x) = 2(\text{floor}(0.9 \sin(x))) + 1.$$

f3: 1, func, sin, .9, *, 2, func, floor, 2, *, 1, +

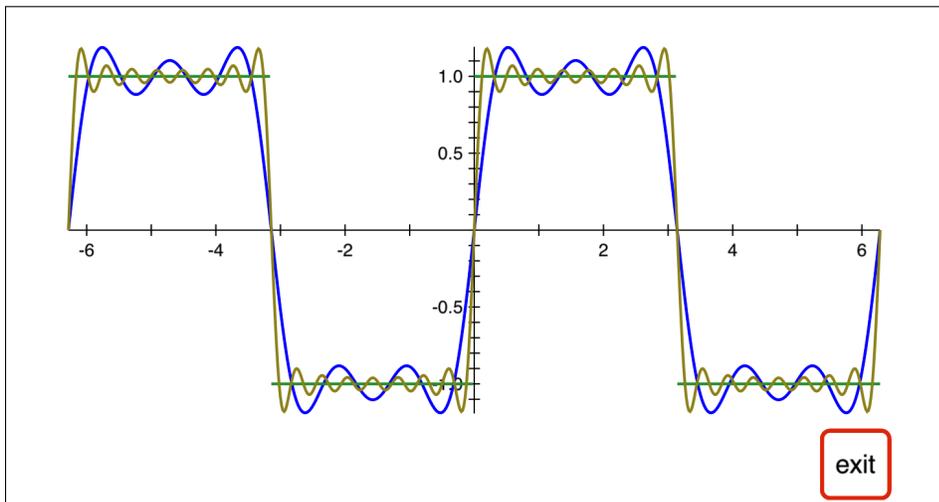
Add the graph of the square wave to the $n = 5$ plot.

7, func, π , -2, *, π , 2, *, 3, add plot



To look at the convergence of the Fourier series, add another plot with $n = 15$.

15, enter, 0, sto, π , -2, *, π , 2, *, 2, add plot



Because we added a plot of function 2 to a list of plots that already had another plot of function 2, trying to zoom in or zoom out from this plot will lose the first function 2 graph. If we wanted the zoom keys to keep all the graphs, we could have defined two different functions for the $n = 5$ and $n = 15$ cases and plotted them with different function numbers.

The reason for plotting the $n = 5$ case before adding the square wave plot is that when adding plots, the first plot sets the viewing window, so plotting the square wave first would cut off the $n = 5$ and $n = 15$ graphs whenever they were above $y = 1$ or below $y = -1$.

Another option is to use the `ymin` and `ymax` keys to explicitly set the minimum and maximum y -values for the graph viewing window, instead of letting the plot function choose them based on the function values encountered during the plot. Then the plots could be added in any order.