Boundary-Value Differential Equation

Use ode called from the solve function for a boundary value problem.

The ode key solves ordinary differential equations, but only initial value problems. To solve a boundary value problem instead, we can create a function with a root that gives the solution and then use the solve function to find that root.

The boundary value problem is

$$y'' = -y, \quad y(0) = 1, \quad y(1) = 0.3$$

For a function with a root that gives the solution to this boundary value problem, define f1(x) to be the value of $y(1)$ after starting with $y(0) = 1$ and $y'(0) = x$, then subtract 0.3, the value of $y(1)$ that we want.

f0: 0, rcl, chs

f1: 7, func, 1, sto, 1, enter, 0, sto, 2, enter, 0, enter, 1, enter, 0, ode, 0.3, −

f0 is the right-hand-side of the d.e., $f(x, y, y') = -y$

f1 sets up the input values for the ode function, solves an initial value problem, then subtracts 0.3

1, sto	(store the current x in register 1 as the initial $y'(0)$)
1, enter, 0, sto	(initial $y(0) = 1$)
2, enter	(order = 2 for the d.e.)
0, enter, 1, enter	(solve the d.e. from 0 to 1)
0, ode	(0 is the function number for the d.e. right-hand-side f0)
0.3, −	(subtract 0.3 from the final $y(1)$)

Since each call to f1 involves solving an ode, function f1 will be slower to evaluate than a function that just does a simple formula. For this reason, we may want to do a few evaluations of f1 by hand first, so we can give the solve function a good starting point that is not far from the root.

30, fix, −0.1, enter, 1, f_n	f1(−0.1) = 0.156155207387350066735686375280
−0.3, enter, 1, f_n	f1(−0.3) = −0.012138989574229234594814089046
−0.5, enter, 1, f_n	f1(−0.5) = −0.180433186535808535925314553372

So we can use −0.3 as the starting point for the solv key to find a root of f1(x) = 0. The maximum running time may need to be increased, using the time key.

−0.3, enter, 1, solv

This gives the root, −0.285574084200894338127940150882, which should be the right initial condition for $y'(0)$ so that the solution hits $y(1) = 0.3$.

Save this value in register 11, then use ode to check that this initial condition hits $y(1) = 0.3$.

7, func, 11, sto, 1, enter, 0, sto, 11, rcl, 1, sto, 2, enter, 0, enter, 1, enter, 0, ode

That gives 0.300000000000000000000000000000.

Another check on the root is to use the analytic solution to this example boundary value problem and solve for the exact value of the root, $(0.3 - \cos(1))/\sin(1)$. This value minus register 11 is about 8.687e−53.