To do a sum with only prime terms, start by defining the prime counting function.

In number theory the prime counting function $\pi(n)$ gives the number of primes less than or equal to n.
The factorization key (fctr) from screen 6 can be used to compute $\pi(n)$. When we do

k, fctr

The function returns k in the y-register on the stack and a factor, usually the smallest, is displayed in the x-register. Then doing a division removes the factor just found and the fctr function can be used again to continue the factorization.
For computing $\pi(n)$, we can exchange x and y, divide, and then apply the floor function. This will give 0 if n is composite and 1 if n is prime, since only when n is prime will x and y be equal.
Define function f2 to sum this function from 2 to n, and the result will be $\pi(n)$.

f1: fctr, x↔y, /, 2, func, floor

f2: 7, func, 2, x↔y, 1, enter, 1, sum

To test this, try

1e5, enter, 2, $\mathrm{f_n}$

giving 9592, the number of primes up to 100,000.

Now approximate this sum where p runs over all the primes.

$$S = \sum_p \frac{1}{p^2} = \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{11^2} + \cdots$$

Modify function f1 in the example above to return $1/j^2$ if j is prime and zero if j is composite.

f1: clrf, 1, func, 1, sto, x^2, 1/x, 1, rcl, 6, func, fctr, x↔y, /, 2, func, floor, *

Sum this series out to 10^2, 10^3, 10^4, and 10^5 terms, saving the results in registers 21, 22, ...

e2, enter, 2, $\mathrm{f_n}$, 21, sto	0.45042878826...
e3, enter, 2, $\mathrm{f_n}$, 22, sto	0.45212043024...
e4, enter, 2, $\mathrm{f_n}$, 23, sto	0.45223760433...
e5, enter, 2, $\mathrm{f_n}$, 24, sto	0.45224661779...

It looks like 10^2 terms gave 2 digits correct, 10^3 gave 3 digits, 10^4 gave 4 digits (almost 5). We might guess that 10^5 terms has 5 (maybe 6) digits correct.
Press on to 10^6 terms (you might have to increase the allowed running time: 6, func, 600, time).

e6, enter, 2, $\mathrm{f_n}$, 25, sto	0.45224735226...

The convergence is a bit slow. If we want more digits, one possibility is to try an extrapolation method (also called acceleration).

This is the Aitken acceleration formula:

$$a_n = x_{n+2} - \frac{(x_{n+2} - x_{n+1})^2}{x_{n+2} - 2\,x_{n+1} + x_n}$$

This will not always improve the convergence of a sequence, but when the values in the x-sequence are converging linearly, Aitken's a-sequence should be more accurate.

We have five approximations to the sum, using 10^k terms with $k = 2, 3, 4, 5, 6$. Call this the x-sequence and use Aitken's formula for the first three, then the middle three, then the last three. That will give three more approximations.

f3 will take 3 inputs, x_n, x_{n+1}, x_{n+2} and return a_n. See the Double Sum example for a fancier function that applies the Aitken formula.

f3: 1, func, 13, sto, roll, 12, sto, roll, 11, sto, 13, rcl, 12, rcl, -, x^2,
13, rcl, 12, rcl, 2, *, -, 11, rcl, +, /, chs, 13, rcl, +

Use f3 with registers 21,22,23, then with 22,23,24, then with 23,24,25 to get three more approximations to the sum. Put those three results into registers 31,32,33.

21, rcl, 22, rcl, 23, rcl, 3, f$_\mathrm{n}$, 31, sto   0.45224632459...

22, rcl, 23, rcl, 24, rcl, 3, f$_\mathrm{n}$, 32, sto   0.45224736891...

23, rcl, 24, rcl, 25, rcl, 3, f$_\mathrm{n}$, 33, sto   0.45224741742...

The last two of these agree to about 7 digits, a bit better than the original sequence gave us. We could even try Aitken's formula again on these three new approximations.

31, rcl, 32, rcl, 33, rcl, 3, f$_\mathrm{n}$, 41, sto   0.45224741978...

This value and the third Aitken approximation above would both round to 0.45224742, so maybe we have increased our accuracy to 8 digits.

The primes are a bit irregular in their spacing, and this may mean the convergence of the series above is not smooth enough for Aitken to help very much.

Compare a related but simpler series.

$$S = \sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots$$

Change f1 to this new function, and change f2 to start the sum at 1 instead of 2.

f1: clrf, 1, func, x^2, 1/x

f2: clrf, 7, func, 1, x↔y, 1, enter, 1, sum

For cases like f2 where the new function is almost the same as the previous definition, it can be quicker to use the editing keys on screen 6 to change the first 2 to a 1.

|← , →, bksp, 1, ←

e2, enter, 2, $\mathrm{f_n}$, 51, sto   1.63498390018...
e3, enter, 2, $\mathrm{f_n}$, 52, sto   1.64393456668...
e4, enter, 2, $\mathrm{f_n}$, 53, sto   1.64483407184...
e5, enter, 2, $\mathrm{f_n}$, 54, sto   1.64492406689...
e6, enter, 2, $\mathrm{f_n}$, 55, sto   1.64493306684...

The last two partial sums agree to about 5 significant digits, a bit less than with the sum over only the primes. Now try the Aitken formula again.

51, rcl, 52, rcl, 53, rcl, 3, $\mathrm{f_n}$, 61, sto   1.64493456785...

52, rcl, 53, rcl, 54, rcl, 3, $\mathrm{f_n}$, 62, sto   1.64493407184...

53, rcl, 54, rcl, 55, rcl, 3, $\mathrm{f_n}$, 63, sto   1.64493406689...

The last two agree to about 8 digits. Applying the Aitken formula to those gives

61, rcl, 62, rcl, 63, rcl, 3, $\mathrm{f_n}$, 71, sto   1.64493406684830...

This agrees to about 10 significant digits with the third value before.

We can't be sure that the agreement in the digits of these various Aitken values really gives the correct value of the sum. A conservative guess might be 8 correct digits, since the last three values we found all agree that far, and maybe we have as many as 10 digits, looking at just the last two.

For this sum, the exact value is known to be $\pi^2/6$, so we can check to see if Aitken actually produced as much accuracy as it seems.

$\pi^2/6 = 1.64493406684822...$

So in this case we have done slightly better than the optimistic guess of 10 digits, and extrapolation has more than doubled the number of accurate digits given by the partial sums.

It appears that because the primes appear at irregular intervals, Aitken extrapolation doesn't help as much in the first example as it does in the second.

Compare this second example with the first example on the Infinite Sums page. That page shows how to get over 50-digit accuracy for this second sum by using the Euler-Maclaurin formula. That method needs for the function $f(n)$ defining the terms in the sum to be a smooth function that can be extended to $f(x)$ for real values of x, so we can evaluate an integral and several derivatives of $f(x)$ in the Euler-Maclaurin formula.

So $f(n) = 1/n^2$ extends to a nice function $f(x) = 1/x^2$ that we can integrate and differentiate, but the summation over only the primes p is the same as

$$\sum_{n=2}^{\infty} f(n), \quad \text{where} \quad f(n) = \begin{cases} 1/n^2, & \text{if } n \text{ is prime} \\ 0\,, & \text{if } n \text{ is composite} \end{cases}$$

That function does not extend to a smooth function over the reals, so there is no easy way the apply the Euler-Maclaurin formula for the prime sum.