

This file extends the FM_User_Manual.txt file for the files that support the FM rational arithmetic package. All of these programs use the 3 basic files discussed there:

1. fmsave.f95 Module for FM internal global variables
2. fm.f95 Subroutine library for multiple-precision operations
3. fmzm90.f95 Modules for interfaces and definitions of derived-types

These are the files included for rational arithmetic.

4. fm_rational.f95 The routines and interfaces for rational operations
5. TestFMrational.f95 Test program for the FM rational routines
6. SampleFMrational.f95 Small sample program using fm_rational

After the first 3 files have been compiled and the program TestFM.f95 has been compiled and run to verify the basic FM package has been installed successfully, as shown in FM_User_Manual.txt, files 4 through 6 can be compiled and run.

The rational arithmetic operations are made available in the user's program by putting
 use fm_rational_arithmetic
at the top of routines using rational arithmetic, and the multiple precision rational variables are declared as
 type (fm_rational)

Here are sample commands to run the programs in files 5, and 6. These are for the gfortran compiler on a Windows PC. Macs are very similar, as are other compilers. See FM_User_Manual.txt.

```
gfortran fm_rational.f95 -c -O3
```

```
gfortran TestFMrational.f95 -c -O3
```

```
gfortran fmsave.o fm.o fmzm90.o fm_rational.o TestFMrational.o -o TestFMrational.exe  
./TestFMrational
```

```
gfortran SampleFMrational.f95 -c -O3
```

```
gfortran fmsave.o fm.o fmzm90.o fm_rational.o SampleFMrational.o -o SampleFMrational.exe  
./SampleFMrational
```

TestFMrational is a program that tests all the rational operations defined in the FM rational arithmetic package. Run it first to make sure that the rational package has been successfully installed. At the end of the run, it should say:

```
499 cases tested. No errors were found.
```

SampleFMrational is a small program showing how to do some rational calculations. It finds all

the rational roots for a polynomial, then does several examples involving solution of linear systems or computation of inverse matrices.

Since problems involving matrices with exact integer or rational entries are common, subroutines for solving linear systems and finding inverse matrices are included in the basic rational package.

SampleFMrational compares the exact rational results with 50-digit approximations from the corresponding FM arithmetic routines.

Timing comparisons show that for the common case of a linear system with fairly small integer coefficients, the exact rational solution can be found faster than the 50-digit FM solution.

The results are saved in the file SampleFMinterval.out.

The basic arithmetic operations are provided, along with assignments and logical comparisons. The functions provided are ones where the function value is an exact rational for all rational input values. Here are the functions that are available for the fm_rational type variables:

abs
ceiling
dim
dot_product
floor
int
is_unknown
matmul
max
maxloc
maxval
min
minloc
minval
mod
modulo
nint
product
rational_approx
rational_numerator
rational_denominator
sum
to_fm
to_fm_rational
to_im
to_zm
transpose

These are the linear algebra routines in this file (see the documentation at the top of each routine for explanation of each of the arguments to the routines).

subroutine rm_lin_solve(a,x,b,n,det) Solve a real $n \times n$ linear system of equations.

subroutine rm_inverse(a,n,b,det) Inverse matrix for a real $n \times n$ matrix