```
Sample 1.  Find all rational roots:  f(t) = 21*t**5 + 43*t**4 - 113*t**3 - 46*t**2 + 49*t + 10 = 0

   Exact rational root found:
      2 / 3
   Exact rational root found:
     -5 / 7



Sample 2.  Solve four n x n linear systems having small integer coefficients.


   fm_lin_solve approximate solution for   25 x  25 system in        0.00 seconds.
             Determinant =
      1.327529583083225364458982193488593754560000000000M+39
             x(1) =
      9.308355168321515846678156296025877808520047438566 7M-1

   rm_lin_solve       exact solution for   25 x  25 system in        0.00 seconds.
             Determinant =
      1327529583083225364458982193488593754560 / 1
             x(1) =
      6178558427896223996632876730585356772 9 / 6637647915416126822294910967442968772 8


   fm_lin_solve approximate solution for   50 x  50 system in        0.02 seconds.
             Determinant =
      1.176718434365915622275285278356032985105360615516 3M+96
             x(1) =
      3.530304932951954756162970787593588079950728077757 5M+0

   rm_lin_solve       exact solution for   50 x  50 system in        0.03 seconds.
             Determinant =
      11767184343659156222752852783560329851053606155163488895788435691668352034065582132074616350
     72576
   /
      1
             x(1) =
      25963593084609330776709791442696628826847720293700388633522222228150017901730670155927899716
     9577
   /
      73544902147869726392205329897252061569085038469771805598677723072927200212909888325466352192
     036


   fm_lin_solve approximate solution for   75 x  75 system in        0.05 seconds.
             Determinant =
      8.645800959388430446649717686814140987436804358415886603821993284760952540542685644050822429 3
             x(1) =
      2.975013288125485298485583129319297245128968016434 5M+0

   rm_lin_solve       exact solution for   75 x  75 system in        0.16 seconds.
             Determinant =
      86458009593884304466497176868141409874368043584158866038219932847609525405426856440508224293
     35653020358721559861396871472658105785761645677590715189064805566 29
```

```
/
   1
          x(1) =
   257213727406686498462824833774024211604248291870055100029723821316520464021761646135578710 69
  305903893387906636887231088638588764503597021353042139029482107498 38
/
   864580095938843044664971768681414098743680435841588660382199328476095254054268564405082242 93
  356530203587215598613968714726581057857616456775907151890648055662 9


fm_lin_solve approximate solution for  100 x 100 system in        0.11 seconds.
          Determinant =
   3.5174880717168312154288707650879389969977895366159M+225
          x(1) =
   1.4924010285917821877921723787844039624726137239165M+0

rm_lin_solve       exact solution for  100 x 100 system in        0.53 seconds.
          Determinant =
   3517488071716831215428870765087938996997789536615854750514857724292019134076993170636981927 6
  7302232865457043689646045990103063906833182152015859754646818585192817849565187196779898771 02
  0751389043164710859177487594752357823235 6
/
   1
          x(1) =
   2624751408144761708710996152595220130890324166423487537281127353438736390740658256340233818 6
  4614895800810562306813250677096469657768716271445304361600757680213061831296633320648547524 04
  8144684289047771839060486901110843302712 9
/
   1758744035858415607714435382543969498498894768307927375257428862146009567038496585318490963 8
  3651116432728521844823022995051531953416591076007929877323409292596408924782593598389949385 51
  0375694521582355429588743797376178911617 8


Sample 3.  Solve four n x n linear systems, this time having non-integer rational coefficients.


fm_lin_solve approximate solution for   10 x  10 system in        0.00 seconds.
          Determinant =
   9.7147804132661796673973639165495117524247426918594M+10
          x(1) =
   7.1366229035259644464643707839561989917846508277142M+0

rm_lin_solve       exact solution for   10 x  10 system in        0.00 seconds.
          Determinant =
   9488975856813825989769837906769 / 97675659697423500000
          x(1) =
   6771924243074246309969093137346 5 / 9488975856813825989769837906769


fm_lin_solve approximate solution for   20 x  20 system in        0.00 seconds.
          Determinant =
   1.5162699369855124181273199328179802380674602563495M+32
          x(1) =
   1.2748020093337303573759042602207355000203323897259M+1

rm_lin_solve       exact solution for   20 x  20 system in        0.01 seconds.
```

```
          Determinant =
   339883043503563478015700820384516569188258785952935656591479377404799819210609929074866884521525140105573756356537749326446418195880575661
/
   224157345082817516749955016639538357405096672178353347068315452060933602003289704170835148800000000000000000
          x(1) =
   433283586796806410007896993689250451409807147323621834488109282323257667909442343394380099161252455038728225154490422759913896027543318342 4
/
   339883043503563478015700820384516569188258785952935656591479377404799819210609929074866884521525140105573756356537749326446418195880575661


fm_lin_solve approximate solution for   30 x  30 system in        0.00 seconds.
          Determinant =
   6.842005154792714970990960369990185533492209522869 7M+54
          x(1) =
   2.059216052003518744627847342190822330395489166806 5M+1

rm_lin_solve       exact solution for   30 x  30 system in        0.03 seconds.
          Determinant =
   263108183184130612110388295436698480111583013501496866560240831677772524171517126223236174816657457954728303751061638993870713182884707419544377033180111200963380565625354577030627170620288282418830487186167768839280575598250069771752349794317817233687840862253087825188652664004242949308425551011476812544811815850770615 3
/
   384548355681707643812312242103300021059830203020733029326014945635967751804318893889296890964605293466261274384146998284130814142204386683770367047728376328065796495536951502139875232590742787430557265110011302455345757605068800000000000000000000000000000000000000000000000
          x(1) =
   162538978267873211588525702377068010301301588942617372550542785588049987502230275786661232425085405031455315004059690630227418308188374901422772050069583809976134723757108950737497387688732712536070444320304237278182169655700256037520297763352990943378312093008748523561332850721461268992563912874760770784933952740247801060
/
   789324549552391836331164886310095440334749040504490599680722495033317572514551378669708524449972373864184911253184916981612139548654122258633131099540333602890141696876063731091881511860864847256491461558503306517841726794750209315257049382953451701063522586759263475565957992012728847925276653034430437634435447552311845 9


fm_lin_solve approximate solution for   40 x  40 system in        0.01 seconds.
          Determinant =
   1.010415874467129259135806772353779278523066509826 8M+80
          x(1) =
   2.892474480505461043468209686267495360443700014333 4M+1

rm_lin_solve       exact solution for   40 x  40 system in        0.11 seconds.
          Determinant =
   142028103395623055023201023943177898712747002823017257135292551973014306296691743315751927636301104603731398268475744732035479178657889608496686533862732724301259110447323632054486623800015292475590766687768736912793088160247519604820033093238305049057373686114441427511335805947666977123681151831606040446801081372702598715718807067022091419527479650637814565671852467958305316923245615721857123751536854659580577849153475536993428345577533498404700955336696818533970943671897877469475179792329288510164145122017790047407915402446211972932439654106798679
/
   140564006350875575095483583560561969255989134830438473890789833737295011455414853537604106 04
```

```
   500958167765094782898009145354771282431132848534987952919144176200385307721037353308598241893
   957438434597616444217511900736415954735880586153902228126832468527642075816442416921279259506
   219620951302183618568576383592195866551621163275364984202820428998750477045029359573084566910
   499837225263597246313683582685339650873592161239040000000000000000000000000000000000000000000
   00000000000
           x(1) =
    534056463962359914807929326751741689154959143041997525343564470210657302709217012627860555 74
   819565756472969243473977430479958767814138684784208342940301806453698270410525602976052495561
   858779241934496568912879610448330364360688750335147005567155035596251259999627630131463509903
   307912032325348702591842389429409254164557302347483090066418612303862436261954117179164963327
   365054009614292681927325384512241036651012504330653146936915413850878749507407168880481267592
   729542412933131067915924268893565918644677258636908785863770527656617075931669041609326201535
  /
    184636534414309971530161331126131268326571103669922434275880317564918598185699266310477505 92
   719143598485081774901846815164612293225525649104569249402155254159163684358152072167083261094
   001988021826799669409935798663101460832177548626604302120979656377458579194877385576473654773
   196707026078549738108785258084140578451337833043444918712871884538572354582915893537340820834
   579691200021930043841426087699791105745475120389951819809145684925079354792611124193770586409
   416222677346724071031773373002807506321338865862312706163029002318007556481217155033883828 27



Sample 4.  Examine determinants of several small Hilbert matrices.



  fm_inverse approximate inverse for    1 x   1 matrix in       0.00 seconds.
           Determinant =
     1.0000000000000000000000000000000000000000000000M+0

  rm_inverse       exact inverse for    1 x   1 matrix in       0.00 seconds.
           Determinant =
     1 / 1


  fm_inverse approximate inverse for    2 x   2 matrix in       0.00 seconds.
           Determinant =
     8.3333333333333333333333333333333333333333333333M-2

  rm_inverse       exact inverse for    2 x   2 matrix in       0.00 seconds.
           Determinant =
     1 / 12


  fm_inverse approximate inverse for    3 x   3 matrix in       0.00 seconds.
           Determinant =
     4.6296296296296296296296296296296296296296296296M-4

  rm_inverse       exact inverse for    3 x   3 matrix in       0.00 seconds.
           Determinant =
     1 / 2160


  fm_inverse approximate inverse for    4 x   4 matrix in       0.00 seconds.
           Determinant =
     1.6534391534391534391534391534391534391534391534392M-7

  rm_inverse       exact inverse for    4 x   4 matrix in       0.00 seconds.
```

```
          Determinant =
     1 / 6048000


   fm_inverse approximate inverse for    5 x   5 matrix in        0.00 seconds.
          Determinant =
     3.749295132515087163613240710746379680620043431834 8M-12

   rm_inverse       exact inverse for    5 x   5 matrix in        0.00 seconds.
          Determinant =
     1 / 266716800000



Sample 5.  Examine determinants of several larger Hilbert matrices.



   fm_inverse approximate inverse for   10 x  10 matrix in        0.00 seconds.
          1 / Determinant =
     4.620689394791469131629562883903627872698368000000 0M+52

   rm_inverse       exact inverse for   10 x  10 matrix in        0.00 seconds.
          Determinant =
     1 / 4620689394791469131629562883903627872698368000000000000

   fm_inverse inverse matrix largest relative error was in row  1 column 10.  Error = 1.08884M-50


   fm_inverse approximate inverse for   20 x  20 matrix in        0.03 seconds.
          1 / Determinant =
     2.377454716768534509091644243427616425743050878213 6M+225

   rm_inverse       exact inverse for   20 x  20 matrix in        0.01 seconds.
          Determinant =
     1
   /
     2377454716768534509091644243427616440175419837753486493033185331234419759310644585187585766 8
    1657377344056575986726555897176563841971079330338658232414981124102355448916615471780963525 77
    9783680000000000000000000000000000000000000

   fm_inverse inverse matrix largest relative error was in row  1 column 20.  Error = 7.09674M-36


   fm_inverse approximate inverse for   30 x  30 matrix in        0.09 seconds.
          1 / Determinant =
     2.939832810239552450584866697140044640136220325634 0M+518

   rm_inverse       exact inverse for   30 x  30 matrix in        0.06 seconds.
          Determinant =
     1
   /
     2939832810239552450554454149671457662704709097017453790237850863133608374886837254022244236 7
    0871543240867850801733611107677638030185935357884574159309950862445430661887957338647852712 38
    5421925367043913854648814370216926583441155883157602140414022772120159075252070534506237128 21
    4455820882780530921346148916982305134960099240772086154178279059855503658572588997750723108 28
    3007463683509421712230491766353997894562500785567662805738963174251576696435327803921719623 68
    0000000000000000000000000000000000000000000000000000000000
```

```
fm_inverse inverse matrix largest relative error was in row  1 column 30.  Error = 1.15448M-20


fm_inverse approximate inverse for   40 x  40 matrix in        0.22 seconds.
          1 / Determinant =
   9.115018598342165363069643960412579234996821323342 3M+931

rm_inverse       exact inverse for   40 x  40 matrix in        0.17 seconds.
          Determinant =
   1
/
   911484453175662587512648766997547325273708411547340528864499267335568114794089121560598423 34
  588941578780273098553144167949448489528336803141772953236332459194497243601609507296388946266
  233062528595411175820228799938417775022913338714288588552485278925773157515869431921894493828
  321447577822636684274663059529149740493580999891430869202808515756322607727305994484227852766
  479158044297125202406142856301182540876335667576022316556769150079984456601875532363261652353
  495277151420824528221223209590100339041009139702891426368118406174050396400813767873460036972
  335023242261321455435085108585664753779398559394807080194265190638932894157867287281594472593
  028619718609003524355718945963579220161775267375998926853659378105451672709373388656591062091
  850744065803164410251281367091228073142835765650074817926193321512072501454051242014007288414
  616186752903472586959093760000000000000000000000000000000000000000000000000000000000000000000
  000

fm_inverse inverse matrix largest relative error was in row  1 column 40.  Error = 2.18693M-5
```

Sample 6. Find four n x n inverse matrices, having random 2-digit numerators and denominators.

```
fm_inverse approximate solution for   10 x  10 system in        0.00 seconds.
          Determinant =
   1.853949586993735438330435895641505304357203050113 4M+8

rm_inverse       exact solution for   10 x  10 system in        0.00 seconds.
          Determinant =
   1740747236392189345058845844768282633080044232800016068193142549169318416453484615517
/
   938939898152727597309169899273586151529445805583698871243572403240960000000 0


fm_inverse approximate solution for   20 x  20 system in        0.01 seconds.
          Determinant =
   5.392806515777939405404239150393611972764924252787 2M+20

rm_inverse       exact solution for   20 x  20 system in        0.07 seconds.
          Determinant =
   1327927732825949030386637954140630291236882545696321744515614196144510103346525711376742968 5
  694219324326632931280963016657142001176436426993024331776714164411323431060807954586977229376
  453941883460980104478546571042899152544485585258945569765874757671326584914552844769154935325
  03363
/
   246240566751427163023598640371612804110779022946407182309627144535294404290125215046912246 14
  209594203732557526955888802011239876236493895217578726471537030422375074102299064286679434745
  796572091839228145172819820469159626523626648371200000000000000000000000000000
```

```
fm_inverse approximate solution for   30 x  30 system in       0.05 seconds.
          Determinant =
  -1.642017061408180484684699695455908519358821265428 4M+33

rm_inverse       exact solution for   30 x  30 system in       0.45 seconds.
          Determinant =
  -7050405418300002154527060716666524350732793757392398563238246478827272961328105296482575858 1
  2957081283672697179499769177354237441737806727931469123401725283365282064531461991411960191 42
  5401158427313774101488920621651430902244121784730465242544016431980940650106681295535486161 99
  2946769905719393976107264116651900816642315127212364190163899986353333135382036236912803966 44
  1680446131901150262384857437104194801293099974159901531691808346107740898098318733049083834 12
  5826503131441056648483597711328076173767668293907542510935279407360011259895385569283293439
/
   4293746748437334614050368956071188526116070920401385745000677460818207357877801422518896492 9
  4671760908319100648072919976881822303021819800901458422114092609421918421391265925240173235 48
  8968299733990923583803715320914538176424790741811352857717420821023757297195046443701606060 54
  5175846114850479795597465280481528785823116001851162819096676497598699323884324803149812987 78
  6098132385800567906544390315254161187645411968956044016366374393388637867056381052486688839 14
  0172820738211840000000000000000000000000000000000000000000000000000000


fm_inverse approximate solution for   40 x  40 system in       0.11 seconds.
          Determinant =
   3.996711083480841563722794811822186036491347579304 4M+50

rm_inverse       exact solution for   40 x  40 system in       1.60 seconds.
          Determinant =
   5679421855071846991606466203061726402262753970256293251321853854226774316373477807745210712 9
  7029378575522066592377462857512971853681455198582638871300290933514607677772763575786524184 70
  9652687088819033776095687990507718343651463966534092757124129926675780652553478569521167339 96
  9651102448462085175247395808296813400637454819257505579682203070366457528502719884782450732 04
  8992549619002884867773252415608302885696912669762391512917816363782963142197430862363200909 96
  5270742917546798399043409662199673640553253447026600876280839110042091323246232970117049060 44
  0677533264372536884267144931101236710237462475151682966260974007283339248647600848655383600 95
  4723690170588118445620834840673871374649866933912289940193057586221170032746139506537794489 03
  2709224986534175264141896298674071460041088515482004887832780229552265388737422 01
/
   1421023870988815143081681016450412074216914709788388250915886061812073565952170067349167055 9
  7322222656212444446684988066600938291077344893856982929841982946763506571371510284078018136 76
  8692564313049344984094800971226818057254176074531889710343792856569407320701231897739039602 05
  9271217655140422385344221648448857198332473269788311841803007271864885453152315148094393015 59
  3086642477010833782209717308507322946730810603956207250663519743194778419673271982577954023 20
  3200279346919132364372476411657630765159116616155161042460249720814017044832288396812567123 67
  5068067201804749791335087231004797871899313299483966918982719539977502965928082585133333504 38
  1580187910900150500025457171984279617776840314220056649138176000000000000000000000000000000 00
  0000000000000000000000000000000


All results were ok -- no errors were found.
```