

Case 1. Call fm_secant to find a root between 1 and 2
for $f(x) = x^{**}2 - 3$.
Use kprt = 0, so no output will be done in the routine, then
write the results from the main program.

A root for function 1 is 1.732050807568877293527446341506

Case 2. Find a root between 6 and 7 for $f(x) = x * \tan(x) - 1$.
Use kprt = 1, so fm_secant will print the result.

fm_secant. Function 2 11 iterations.
Estimated relative error = 7.767234M-58, Root:
6.4372981791719471203626398510256332453217341714480M+0

Case 3. Find a root between 1 and 5 for $f(x) = \text{gamma}(x) - 10$.
Use kprt = 2, so fm_secant will print all iterations,
as well as the final result.

fm_secant. Begin trace of all iterations.

j = 0	$f(ax) = -9.0000000000M+0$	x:	1.00M+0
j = 0	$f(bx) = 1.4000000000M+1$	x:	5.00M+0
j = 1	$f(x) = -8.6068421859M+0$	x:	2.5652173913043478260869565217391304347826086956522M+0
j = 2	$f(x) = -6.7051471157M+0$	x:	3.4921840811950674768726054094643443433586793877230M+0
j = 3	$f(x) = 4.5186929492M+2$	x:	6.7605566635344553427953681683199185886633746295652M+0
j = 4	$f(x) = -6.5259077772M+0$	x:	3.5399733101293708712941683890968476230216022770789M+0
j = 5	$f(x) = -6.3422248053M+0$	x:	3.5858228954424667483373209259448565775174368926113M+0
j = 6	$f(x) = 2.1049716711M+1$	x:	5.1689221582444679282516848043680371219598475726118M+0
j = 7	$f(x) = -4.3466346198M+0$	x:	3.9523676110766884825207209243892885805725532099791M+0
j = 8	$f(x) = -2.6325150260M+0$	x:	4.1605832733470755491853842973219340298745655186153M+0
j = 9	$f(x) = 1.3192340241M+0$	x:	4.4803572620082733981333836380656569924036270263122M+0
j = 10	$f(x) = -2.2137831189M-1$	x:	4.373605360378087300062328166200630474362567876779M+0
j = 11	$f(x) = -1.5402542083M-2$	x:	4.3889450764262963809325836229953951301411172475795M+0
j = 12	$f(x) = 1.9744111815M-4$	x:	4.3900921561153248909886245693114366521619353797757M+0
j = 13	$f(x) = -1.7323119072M-7$	x:	4.3900776381063320867951914980063629286730825522973M+0
j = 14	$f(x) = -1.9461226903M-12$	x:	4.3900776508329989159958480043129330063285601971190M+0
j = 15	$f(x) = 1.9182672042M-20$	x:	4.3900776508331418921725660042962397803248267899607M+0

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j = 16   f(x) = -2.1241838752M-33           x:
        4.3900776508331418921711567071874581130384786044923M+0
j = 17   f(x) = -2.3185368313M-54           x:
        4.3900776508331418921711567071874582690963097189549M+0
j = 18   f(x) = -1.0000000000M-77           x:
        4.3900776508331418921711567071874582690963097189549M+0
j = 19   f(x) = -1.0000000000M-77           x:
        4.3900776508331418921711567071874582690963097189549M+0

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fm_secant. Function 3 19 iterations.
 Estimated relative error = 1.138932M-57, Root:
 $4.3900776508331418921711567071874582690963097189549M+0$

Case 4. Find a root between 1 and 2 for $f(x) = \text{polygamma}(0, x)$.
 Use kprt = 1, so fm_secant will print the result.

fm_secant. Function 4 11 iterations.
 Estimated relative error = 3.420833M-57, Root:
 $1.4616321449683623412626595423257213284681962040064M+0$

Case 5. Find a root near 3.1 for $f(x) = \cos(x) + 1$. (Double root)
 Use kprt = 2, so fm_secant will print the iterations.

fm_secant. Begin trace of all iterations.

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j = 0   f(ax) = 8.6484972672M-4           x:
        3.10000000000000000000000000000000000000000000000000000000000000M+0
j = 0   f(bx) = 1.7052242052M-3           x:
        3.20000000000000000000000000000000000000000000000000000000000000M+0
j = 1   f(x) = 1.0422702438M-2           x:
        2.9970875783570827008391241954412154141991368316547M+0
j = 2   f(x) = 4.8078499092M-3           x:
        3.2396916589497320151463064609294213432799905009368M+0
j = 3   f(x) = 4.6403965569M-2           x:
        3.4474271249802673219623508910459796120747925426733M+0
j = 4   f(x) = 2.7432671392M-3           x:
        3.2156807404865076385409933147079484010368360207527M+0
j = 5   f(x) = 1.7712155432M-3           x:
        3.2011197677104049910687744827986342665736646250163M+0
j = 6   f(x) = 5.4428436058M-4           x:
        3.1745876150836989946718504008020165861243120692951M+0
j = 7   f(x) = 2.2524007549M-4           x:
        3.1628175696799296207951052135294249468006868205820M+0
j = 8   f(x) = 8.3403335932M-5           x:
        3.1545081090839358367174767078372923961650525112318M+0
j = 9   f(x) = 3.2234635163M-5           x:
        3.1496219509866413602496999070622136243836055823282M+0
j = 10  f(x) = 1.2257041924M-5           x:
        3.1465438285944167712048011249784096248313251195505M+0
j = 11  f(x) = 1.5313139121M-3           x:
        3.1446552790200871817410158401485968932236201175305M+0
j = 12  f(x) = 2.4832118180M-3           x:
        3.1465590670175217495090723179227983616801874230198M+0
j = 13  f(x) = 5.0884881537M-9           x:
        3.1415926637667695457830203649682113061534827694136M+0

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j = 14    f(x) =  1.0459100332M-14          x:
         3.1415926535898141566633070137156315280367587416150M+0
j = 15    f(x) =  9.0271675357M-32          x:
         3.1415926535897932384626433832796834275478838739507M+0
j = 16    f(x) =  3.2916898334M-60          x:
         3.1415926535897932384626433832795028841971693993751M+0
j = 17    f(x) =  5.0068598259M-79          x:
         3.1415926535897932384626433832795028841971693993751M+0
j = 18    f(x) =  5.0068598259M-79          x:
         3.1415926535897932384626433832795028841971693993751M+0

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fm_secant. Function 5 18 iterations.

Estimated relative error = 1.591549M-57, Root:
 3.1415926535897932384626433832795028841971693993751M+0

Case 6. Find a root near 3.1 for $f(x) = \cos(x) + 1 - 1.0e-40$.

There are two different roots that agree to about 20 digits,
 so here the convergence is slower.

Use kprt = 1, so fm_secant will print the result.

fm_secant. Function 6 54 iterations.

Estimated relative error = 1.591549M-57, Root:
 3.1415926535897932384767855189032338346851862866172M+0

Case 7. Find a root near 3.1 for $f(x) = \sin(x)^{*}3$. (Triple root)

Use kprt = 2, so fm_secant will print the iterations.

fm_secant. Begin trace of all iterations.

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j = 0    f(ax) =  7.1890948202M-5          x:
         3.100000000000000000000000000000000000000000000000000000000000000000M+0
j = 0    f(bx) = -1.9891226494M-4          x:
         3.200000000000000000000000000000000000000000000000000000000000000000M+0
j = 1    f(x) =  3.4053191807M-6          x:
         3.1265473025109802113579063768052712098830330073810M+0
j = 2    f(x) = -4.6033019682M-3          x:
         3.1277836254965637319671255990161395030696935340751M+0
j = 3    f(x) = -5.0151907695M-3          x:
         3.1265482164119758136089201494691601198372701516572M+0
j = 4    f(x) = -6.6600352394M-7          x:
         3.1415906555792214179684872785638182310056451601383M+0
j = 5    f(x) = -5.0254086750M-11         x:
         3.1415926534390309782133329510918323393079713387829M+0
j = 6    f(x) = -6.6877258692M-23         x:
         3.1415926535897932384624427515034254892529502674081M+0
j = 7    f(x) = -5.0669018059M-43         x:
         3.1415926535897932384626433832795028841971678793046M+0
j = 8    f(x) =  3.3379065506M-79         x:
         3.1415926535897932384626433832795028841971693993751M+0
j = 9    f(x) =  3.3379065506M-79         x:
         3.1415926535897932384626433832795028841971693993751M+0

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fm_secant. Function 7 9 iterations.

Estimated relative error = 1.591549M-57, Root:
 3.1415926535897932384626433832795028841971693993751M+0

